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Jumps in Real-time Financial Markets:
A New Nonparametric Test and Jump Dynamics *

Suzanne S. Lee and Per A. Mykland

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The University of Chicago
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Abstract

This paper introduces a new nonparametric jump test for continuous-time asset pricing models. It distinguishes jump arrival times and realized jump sizes in asset prices up to at intra-day levels. We demonstrate the likelihood of misclassification of jumps in discrete data becomes negligible when we use high-frequency returns. We explore real-time jump dynamics using intra-day U.S. individual equity prices through the test and find empirical evidence that jump arrivals are associated with both prescheduled earnings announcements and unscheduled real-time news releases.

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Financial market evolution is often interrupted by various events such as market crashes, corporate defaults or announcements, central bank announcements, and news releases. These market uncertainties generate significant discontinuities in financial variables. Empirical evidence of such discontinuities, so-called "*Jumps*" in financial markets, has been well documented in recent literature. A number of studies have proved the substantial impact of jumps on financial management, from portfolio and risk management to option or bond pricing and hedging [see Merton (1976), Bakshi, Cao, and Chen (1997, 2000), Bates (1996, 2000), Liu *et al.* (2003), Eraker *et al.* (2003), Naik and Lee (1990), Duffie *et al.* (2000), and Piazzesi (2003)]. It turns out to be important to incorporate jumps into continuous-time asset pricing models because they provide reasonable explanations for financial market phenomena such as the excess kurtosis and skewness of return distributions and the implied volatility smile in option markets. Furthermore, they allow for the management of risk associated with their particular hedging demand, in addition to the diffusive risk to which market participants are usually exposed. In order for investors to manage these two types of risk, it is critical that they be able to distinguish jumps from diffusion.

However, disentangling jumps in continuous-time models is difficult, since we can only observe realized data at discrete times. It becomes a challenge to make an econometric inference for continuous-time jump diffusion models because some discrete data may be from the diffusion part, yet yield exactly the same discreteness as that from the jump part. Incorrectly identified jumps will affect estimation of jump intensity and distribution, misleading investors. In addition, classifying the arrival times and sizes of realized jumps in terms of their relevance to the application may be necessary. A thorough investigation of jump arrival dynamics can help us characterize and forecast future return distributions. It can also deepen our understanding of various stylized facts in individual equity option markets, such as flatter implied volatilities of equity options than index options [see Bakshi, Kapadia, and Madan (2003) and Maheu and McCurdy (2004)].

The purpose of this paper is to propose a new testing device that can identify arrival dates and times of realized jumps and their sizes, providing a model-free tool for investigating the dynamic behavior of jumps up to at intra-day levels. We prove that the new test can precisely distinguish actual jumps from high-frequency data and spurious detection of jumps becomes negligible. Therefore, we show that stochastic jump estimates based on our test are accurate. We specifically illustrate that the probabilities of misclassification becomes zero and provide the convergence rate as well.

Overall characteristics of return distributions can be found as a by-product, since both the signs and sizes of jumps can be obtained whenever jumps are detected. Detecting the sizes and arrival times of jumps is important for dynamic hedging, as well as many other applications [see Naik and Lee (1990), and Bertsimas *et al.* (2001)]. The impact of jumps becomes more crucial as their variance increases, and especially as they increase in size, because hedging error is more likely to exceed one's tolerance level. In other words, the size of jumps determines the degree of market incompleteness. Arrival time detection becomes important for dynamic re-balancing of a derivative hedging portfolio, as explained by Collin-Dufresne and Hugonnier (2001).

Our test is nonparametric, which makes the results robust to model misspecification. There are a few other nonparametric approaches to jumps described in the literature. Aït-Sahalia (2002) suggests a diffusion criterion based on transition density to test the presence of jumps. Bandi and Nguyen (2003) and Johannes (2004) provide consistent nonparametric estimators for jump diffusion models based on the kernel estimation method. Barndorff-Nielsen and Shephard (2006) suggest tests to indicate the presence of jumps over a certain time interval by comparing realized power and bipower variation. Although their methods can identify a jump's presence, because they use two integrated quantities and the integrations contain all possible jumps in the interval, they cannot distinguish how many jumps were present within the interval, when in the interval a jump occurs, or how large the realized jump sizes are. By simulation, we show that our test performs better than or equal to tests by Barndorff-Nielsen and Shephard (2006). Tauchen and Zhou (2005) used Barndorff-Nielsen and Shephard's (2006) test to find realized jump sizes by strictly assuming there is, at most, one jump in a day. However, according to the empirical evidence from our test, it is possible that there can be more than one jump a day in U.S. equity markets. Recently, there is increased interest in this problem by other research groups. A wavelet approach by Fan and Wang (2005) and a test based on a variance swap-replicating strategy by Jiang and Oomen (2005) are also under development.

A number of different parametric inference methodologies have been developed and applied in empirical studies of jumps in continuous-time asset pricing models using time series and cross-sectional data. They include the Implied State Generalized Method of Moments (IS-GMM) [see Pan (2002)], Maximum Likelihood Estimation [see Schaumburg (2001)], the simulation-based Efficient Method of Moment (EMM) [see Andersen *et al* (2002) and Chernov *et al*. (2003)], the Bayesian approach [see Eraker *et al*. (2003)], volatility estimation and different types of jumps under Levy processes [see Aït-Sahalia (2003) and Aït-Sahalia and Jacod (2005)]. All, however, run the risk of incorrect specification for functionals in their chosen models, which is not the case with our nonparametric test.

Another merit of our new test is that it is robust to nonstationarity of the processes, which is a common feature of financial variables.

We conducted an empirical study on real-time jump dynamics in the U.S. individual equity market. It is based on high-frequency returns from four different individual stock prices transacted on the New York Stock Exchange (NYSE) over the period of September 1 to November 30, 2005. Our empirical observations through our new test indicate that most jumps in equity prices arrive with news events. Specifically, we connect jump arrival times we detected with real-time news releases from Factiva, a Dow Jones & Reuters Company. We find that accumulated real-time news about a company from the early morning before the market starts tends to create a jump in the company's stock price around market opening time. Prescheduled news announcements of corporate earnings turn out to be always associated with jumps around the opening time. In addition to prescheduled news events, the majority of jumps were connected with unscheduled news and the magnitudes of jump sizes with unscheduled news were comparable to those with prescheduled ones. This outcome on underlying stocks suggests a foundation for individual equity option pricing models associated with sound fundamentals. Market price of jump risks can be better examined with cross-sectional option data after setting up models with more precise jump dynamics though our test.

The rest of the paper is organized as follows. Section 1 sets up a theoretical model framework for financial variables to test jumps. It defines the test statistics and describe the intuition behind our new test, and derives its asymptotic distribution to provide a benchmark for tests. In Section 2 we discuss both how the rejection region for the test can be determined and the likelihood of misclassifications. Section 3 investigates finite sample performance of the test by simulation. Section 4 presents empirical evidence of real-time jump dynamics in equity prices and the association with real-time news. Finally, we conclude in Section 5. All the proofs are in Section 6.

1 A Theoretical Model for the Test and Its Asymptotic Theory

We employ a one-dimensional asset return process with a fixed complete probability space $(\Omega, \mathcal{F}_t, \mathcal{P})$, where $\{\mathcal{F}_t : t \in [0, T]\}$ is a right-continuous information filtration for market participants, and $\mathcal{P}$ is a data-generating measure. Let the continuously compounded return be written as $d \log S(t)$ for $t \geq 0$, where $S(t)$ is the asset price at t under $\mathcal{P}$. We are interested in testing jumps in the asset returns as follows. The null hypothesis of no jumps in the market is represented as

$$d \log S(t) = \mu(t)dt + \sigma(t)dW(t) \tag{1}$$

where $W(t)$ is a $\mathcal{F}_t$-adapted standard Brownian motion. The drift $\mu(t)$ and spot volatility $\sigma(t)$ are $\mathcal{F}_t$-measurable functions, such that the underlying process is a diffusion that has continuous sample paths. Its alternative hypothesis is given by

$$d \log S(t) = \mu(t)dt + \sigma(t)dW(t) + Y(t)dJ(t) \tag{2}$$

where $dJ(t)$ is a jump-counting process with intensity $\lambda(t)$ and $Y(t)$ is the jump size whose mean is $\mu_y(t)$ and standard deviation is $\sigma_y(t)$, which are also $\mathcal{F}_t$-measurable functions. We assume $W(t)$ and $J(t)$ are independent. Observation of $S(t)$, equivalently $\log S(t)$, only occurs at discrete times $0 = t_0 < t_1 < ... < t_n = T$. For simplicity, this paper assumes observation times are equally spaced: $\Delta t = t_i - t_{i-1}$. This simplified assumption can easily be generalized to non-equidistant cases by letting $\max_i(t_i - t_{i-1}) \to 0$. We also impose the following necessary assumption on price processes throughout this paper:

Assumption 1

$$\textbf{A1.1} \;\; sup_i \;\; sup_{t_i \leq u \leq t_{i+1}} |\mu(u) - \mu(t_i)| = O_p(\sqrt{\Delta t})$$

$$\textbf{A1.2} \;\; sup_i \;\; sup_{t_i \leq u \leq t_{i+1}} |\sigma(u) - \sigma(t_i)| = O_p(\sqrt{\Delta t})$$

Following Pollard (2002), we use O_p notation throughout this paper to mean that, for random vectors $\{X_n\}$ and non-negative random variable $\{d_n\}$, $X_n = O_p(d_n)$, if for each $\epsilon > 0$, there exists a finite constant M_ϵ such that $P(|X_n| > M_\epsilon d_n) < \epsilon$ eventually. One can interpret **Assumption 1** as that the drift and diffusion coefficients do not change dramatically over a short time interval. Furthermore, it allows the drift and diffusion to depend on the process itself. Therefore, this assumption is general enough to cover most of the models that incorporate jumps in continuous-time asset price processes in the literature. It also satisfies the stochastic volatility plus finite activity jump semi-martingale class in Barndorff-Nielsen and Shephard (2004) and the reference therein.

1.1 Definition and Intuition of the Nonparametric Jump Test

In this subsection, we define the jump test statistic T, and address the basic intuition behind our new testing technique. Our discussion in this subsection is on a single test at time t_i. We do not assume there was or was not a jump before or after t_i. Generalization to a global test to determine whether a diffusion model (1) is rejected is straightforward by multiple tests (single tests over available times). The global test is interesting in itself and is also part of our goal. However, our main purpose of this paper is to detect jumps in order to advance our knowledge on underlying mechanism of how these jumps evolve over time. Multiple tests can allow us to extract such information on jump arrival dynamics.

Here, we describe the formulation of the statistic and provide its mathematical definition. Suppose we have a fixed time horizon T, and n is the number of observation in $[0, T]$. $\Delta t = \frac{T}{n}$. Consider a local movement of the process within a window size K. With realized returns in the window consisting of the previous K observations just before a testing time t_i, the time-varying local volatility is estimated based on the realized bipower variation. We then take the ratio of this estimated local volatility to the next realized return to determine whether there was a jump and how large the jump size was. For example, if $\Delta t = 5$ minutes, $t_i = 10:05$ AM and $K = 10$, then we test for a jump by examining the relative magnitude of realized return from $10:00$ AM to $10:05$ AM compared to local volatility estimated with 5-minute returns from $9:10$ AM to $10:00$ AM. Figure 1 also illustrates the construction of the test. Mathematical notation of the test statistics is as follows:

Definition 1 *The statistic $T(i)$, which tests at time t_i whether there was a jump from t_{i-1} to t_i, is defined as*

$$T(i) = \frac{\log S(t_i) - \log S(t_{i-1})}{\widehat{\sigma(t_i)}},$$

where

$$\widehat{\sigma(t_i)} = \sqrt{\frac{1}{K-2} \sum_{j=i-K+2}^{i-1} |\log S(t_j) - \log S(t_{j-1})||\log S(t_{j-1}) - \log S(t_{j-2})|}.$$

The intuition is as follows. Imagine that asset prices evolve over time continuously. A jump arrives in a market at some time, say t_i. If the horizon is short, we would expect the realized asset return to be much higher than usual returns due to purely continuous random innovations. How about the situation when the volatility at that time was also high? Even if there was no jump, if the volatility is high and if we can only observe prices in discrete times, the realized return we observe may also be as high as the return due to a jump. To distinguish those two cases, it is natural to standardize the return by a measure that explains the local variation from only the continuous part of the process. We call this measure *local volatility*[1] in this paper and denote it

[1]Note that in the literature on implied binomial trees for option pricing, the term "local volatility" is used in a

as $\sigma(t_i)$. This idea is incorporated into our test. It compares a realized return at any given time to a consistently estimated *local volatility* using corresponding local movements of returns. More specifically, the ratio of realized return to estimated local volatility creates the test statistic for jumps.

Now, how do we estimate *local volatility*? A commonly used nonparametric estimator for variance in the literature is the *realized power (quadratic) variation*, defined as the sum of squared returns

$$\text{plim}_{n\to\infty}\sum_{i=2}^{n}(\log S(t_i) - \log S(t_{i-1}))^2.$$

Using high-frequency returns within some period just before our testing time, it suggests a variance estimate over that period. However, this well-known variance estimator is unfortunately inconsistent under the presence of jumps in a return process. Alternatively, a slightly modified version called the *realized bipower variation*, defined as the sum of products of consecutive absolute returns

$$\text{plim}_{n\to\infty}\sum_{i=3}^{n}|\log S(t_i) - \log S(t_{i-1})||\log S(t_{i-1}) - \log S(t_{i-2})|$$

has been suggested and has been shown to be a consistent estimator for the integrated volatility, even when there are jumps in return processes [see Barndorff-Nielsen and Shephard (2003) and Aït-Sahalia (2004)]. Despite the intuition that jumps in a process may impact the volatility estimation, it remains consistent no matter how large or small the jump sizes are mixed with the diffusive part of pricing models. Our test is based on this interesting insight. Even if a highly volatile market environment makes the distinguishing of jumps harder, infrequent Poisson jumps will be detected by our procedure pretty accurately, as long as we use high-frequency observations. Notice that the realized bipower variation is used in the local volatility estimation for the denom-

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different manner

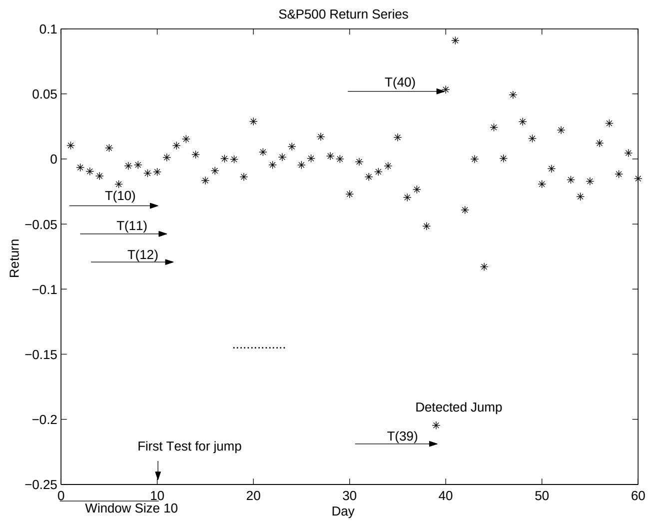

Figure 1: Formation of our new test with a window size $K = 10$

inator of the statistic. This makes our technique immune to the presence of jumps in previous times, especially those that are considered for local volatility estimation. We choose the window size K in such a way that the effect of jumps on the volatility estimation disappears. In the following subsections, we show that T asymptotically follows a normal distribution if there is no jump at testing times, and suggest a criterion to distinguish arrival of jumps based on this asymptotic distribution.

We state our results ignoring the drift. For our analysis using high-frequency data, the drift (of order dt) is mathematically negligible compared to the diffusion (of order $\sqrt{dt}$) and the jump component (of order 1). In fact, the drift estimates have higher standard errors, so that they make the precision of variance estimates decrease if included in variance estimation. We study a simplified version of the model without the drift term, i.e., $\mu = 0$. We also show that the main result we present continues to hold with the non-zero drift term in Appendix 6.1. A modified statistic T_μ for the non-zero drift case is defined, and a corresponding theorem is presented therein as well.

1.2 Under the Absence of Jumps at Testing Time t_i

Suppose our realized return from time t_{i-1} to t_i is from a scalar diffusion process without a drift, as

$$d \log S(t) = \sigma(t) dW(t).$$

The asymptotic null distribution of the jump test statistic T is provided in the following Theorem 1.

Theorem 1 *Let* $T(i)$ *be as in* **Definition 1** *under the null at time* t_i *and* $K = O_p(\Delta t^{\alpha})$*, where* $-1 < \alpha < -0.5$*. Suppose* **Assumption 1** *is satisfied. Then as* $\Delta t \to 0$*,*

$$sup_i |T(i) - \widehat{T}(i)| = O_p(\Delta t^{\frac{3}{2} - \delta + \alpha}),$$

where δ *satisfies* $0 < \delta < \frac{3}{2} + \alpha$ *and*

$$\widehat{T}(i) = \frac{U_i}{c}.$$

Here $U_i = \frac{1}{\sqrt{\Delta t}}(W_{t_i} - W_{t_{i-1}})$ *and* $c = E|U_i| = \sqrt{2}/\sqrt{\pi} \approx 0.7979$.

Proof of Theorem 1: *See Appendix 6.2.*

Theorem 1 states our test statistic $T(i)$ approximately follows the same distribution as $\widehat{T}(i)$. And, we find $\widehat{T}(i)$ follows a normal distribution with mean 0 and variance $\frac{1}{c^2}$ because U_i is a standard normal random variable. In case we absolutely know a priori the absence of jumps in the whole price process, the usage of realized quadratic variation for estimating *local volatility* would yield the same asymptotic distribution. As discussed in our intuition, however, we do not require the absence of jumps in earlier or later times. Therefore, using quadratic variation does not suffice in this case. As also can be noticed, $T(i)$ is asymptotically independent and normally distributed over time; hence, one can easily find a joint asymptotic null distribution of the test statistics for various periods.

1.3 Under the Presence of Jumps at Testing Time t_i

We show in this subsection how the jump test reacts to the arrival of jumps and discuss the choice of window size. The realized return from time t_{i-1} to t_i is now from a scalar jump diffusion, which describes the arrival of jumps in addition to the diffusive market innovation as

$$d \log S(t) = \sigma(t) dW(t) + Y(t) dJ(t).$$

Theorem 2 demonstrates how our test behaves differently when a jump arrives. It specifically shows that as the sampling interval Δt goes to 0, the test statistic becomes so large that we can reject the hypothesis that no jump arrived.

Theorem 2 *Let $T(i)$ be as in* **Definition 1** *under the alternative at time t_i. If $K = O_p(\Delta t^\alpha)$, where $-1 < \alpha < -0.5$, then*

$$T(i) \approx \frac{U_i}{c} + \frac{Y(\tau)}{c\sigma\sqrt{\Delta t}} I_{\tau \in (t_{i-1}, t_i]}$$

and $T(i) \to \infty$ as Δt goes to 0.

Proof of Theorem 2: *See Appendix 6.3.*

The benefit of the bipower variation as a *local volatility* estimator in the denominator of the statistic is that the presence of jumps in earlier times doesn't affect the consistency of volatility estimation. Therefore, our test is robust to earlier jumps in detecting current jumps. This does not imply that we make no use of earlier jumps. One can learn about an earlier jump arrival by doing the same single test at that earlier time. Several single tests over time become a multiple test, which can provide us more information on jump dynamics.

In order to retain the benefit of bipower variation, the window size K has to be large enough, so that the effect of jumps in return on estimating local volatility disappears, but obviously smaller than the number of observations n. The condition $K = O_p(\Delta t^{\alpha})$ with $-1 < \alpha < -0.5$ satisfies this requirement. Therefore, the choice of sampling frequency Δt will determine the window size. For example, if daily data are used in the analysis, $\Delta t = \frac{1}{252}$, and $K = \beta \Delta t^{\alpha}$ with $\beta = 1$, integers between 15.87 and 252 are within the required range. The simulation study in Subsection 3.5 finds that if K is within the required range, increasing K only elevates the computational burden without a marginal contribution. Hence, the smallest integer K that satisfies the necessary condition would be an optimal choice of K. In this example, $K^{opt} = 16$, i.e., the local volatility estimation is based on observations of about three weeks just prior to the test using daily data.

1.4 Selection of Rejection Region

In this subsection, we address the rejection region for our proposed test. In effect, we demonstrate that the suggested rejection region allows us to distinguish jumps more precisely at higher frequencies of observation.

As studied in Theorem 1 and 2, our test statistics present completely different limiting behavior depending on the existence of jumps at testing times. If there is no jump at the testing time, our test statistic follows approximately a normal distribution. But if there is a jump, the test statistic becomes very large. To determine a reasonable rejection region, we raise a question of how large our test statistic can be when there is no jump. Hence, we first study the asymptotic distribution of maximums of our test statistics under the null hypothesis. Such a distribution then guides us to choose the relevant threshold for the test to distinguish the presence of jumps at a testing time. **Lemma 1** states the limiting distribution of the maximums as follows:

Lemma 1 *If the conditions for $T(i)$, K, and c are as in* **Theorem 1** *under the null at time t_i, then as $\Delta t \to 0$*

$$\frac{max|T(i)| - C_n}{S_n} \to \xi,$$

where ξ has a cumulative distribution function $P(\xi \leq x) = exp(-e^{-x})$,

$$C_n = \frac{(2\log n)^{1/2}}{c} - \frac{\log \pi + \log(\log n)}{2c(2\log n)^{1/2}} \ and \ S_n = \frac{1}{c(2\log n)^{1/2}},$$

where n is the number of observations.

Proof of Lemma 1: *See Appendix 6.4.*

In short, the main idea in selecting a rejection region is that if our observed test statistics are not even within the usual region of maximums, it is unlikely that the realized return is from a diffusion process. To apply this result to selecting a rejection region, for instance, we can set a significance level of 1%. Then, the threshold for $\frac{|T(i)|-C_n}{S_n}$ is β^*, such that $P(\xi \leq \beta^*) = exp(-e^{-\beta^*}) = 0.99$. Equivalently, $\beta^* = -\log(-\log(0.99)) = 4.6001$. Therefore, if $\frac{|T(i)|-C_n}{S_n} > 4.6001$, then we reject the hypothesis of no jump at t_i.

2 Misclassifications

In what follows, we define four different types of misclassifications that can occur in our study. We then demonstrate that the probability of such misclassifications becomes negligible at higher frequencies of observation. For a single testing time, say t_i, there can be two kinds of misclassifications. The first kind is when there is a jump in interval $(t_{i-1}, t_i]$, but the test fails to declare its existence. We call this a *failure to detect actual jump (FTD$_i$)* at t_i. The second kind of misclassification is when there is no jump in $(t_{i-1}, t_i]$, but the test wrongly declares there is one. We call this a *spurious detection of jump (SD$_i$)* at t_i. It is usually the case that we do this test n times with n observations. Global extension of these concepts is straightforward. If there are some jumps over the whole time horizon, $[0, T]$, but the test fails to detect any one of them, we call it a *global failure to detect jump (GFTD)*. If there are some returns that are not due to jumps, but the procedure wrongly declares any one of them as due to a jump, we call it a *global spurious detection of jump (GSD)*. We will use the following mathematical notations to explain the above situations. We let A_n be jump times among n observations and B_n be times at which the test declares the presence of a jump. We use J_i (J is for jumps) to denote the event that there is a jump in $(t_{i-1}, t_i]$. Note that $J_i = \{i \in A_n\}$. D_i (D is for declaring jumps) denotes the event that our test declares a jump in $(t_{i-1}, t_i]$. In this case, $D_i = \{i \in B_n\}$. Then following statements holds.

$$\textit{failure to detect actual jump at } t_i \textit{ (local property) } (FTD_i) = J_i \bigcap D_i^C$$

$$\textit{spurious detection of jump at } t_i \textit{ (local property) } (SD_i) = J_i^C \bigcap D_i$$

$$\textit{failure to detect actual jumps (global property) } (GFTD) = \bigcup_{i=1}^{n} \left(J_i \bigcap D_i^C \right)$$

$$\textit{spurious detection of jumps (global property) } (GSD) = \bigcup_{i=1}^{n} \left(J_i^C \bigcap D_i \right)$$

With these new notations, we now generalize the above example using a fixed significance level to any significance level α_n that approaches 0. Alternatively, β_n approaches ∞. In Theorem 3 and Corollary 1, we explicitly show that both the conditional and unconditional probabilities of

GFTD approach 0. They state specifically the convergence rate to show how fast these probabilities converge to 0.

Theorem 3 *Let β_n be the $(1-\alpha_n)^{th}$ percentile of the limiting distribution of ξ in* **Lemma 1***, where α_n is the significance level of the test. Suppose there are N jumps in $[0,T]$. Then, the probability of GFTD is*

$$P(GFTD|N\ jumps) = \frac{2}{\sqrt{2\pi}} y_n N + o(y_n N),$$

where $y_n = (\beta_n S_n + C_n) c\sigma\sqrt{\Delta t}$. Therefore, as long as $\beta_n \to \infty$ slower than $\sqrt{n \log n}$,

$$P(GFTD|N\ jumps) \longrightarrow 0$$

Proof of Theorem 3: *See Appendix 6.5.*

The result above does not depend on model (2), so long as jumps are independent of diffusion part of the model. In particular, prescheduled (deterministic) events, such as earnings announcements, are accommodated. If jumps arrive randomly, as in (2), with constant intensity λ, one can expect the probability to be as in Corollary 1.

Corollary 1 *If the jump intensity is λ and time horizon is from 0 to T, then*

$$E[P(GFTD)] = \frac{2}{\sqrt{2\pi}} y_n \lambda T + o(y_n \lambda T).$$

Theorem 4 also presents a generalized likelihood that *GSD* approaches 0 quickly. The corresponding convergence rate is provided as the significance level α_n become close to 0 or, equivalently, the rejection threshold β goes to ∞.

Theorem 4 *Let* β_n *be as in* **Theorem 3**. *Again, suppose there are N jumps in* $[0, T]$. *Then, as* $\Delta t \to 0$, *the probability of GSD*

$$P(GSD|Njumps) = exp(-\beta_n) + o(exp(-\beta_n)).$$

Therefore, as $\beta_n \to \infty$,

$$P(GSD|N\ Jumps) \longrightarrow 0$$

Proof of Theorem 4: *See Appendix 6.6.*

The immediate consequence of our findings in the previous two theorems is that we can classify actual and spurious jumps precisely so that we obtain an accurate stochastic jump intensity estimator based on our nonparametric test. If the probabilities in Theorems 3 and 4 become negligible, then the likelihood of global misclassification is also negligible, as stated in Theorem 5, below.

Theorem 5 *If* $\widehat{\Lambda}(T)$ *is the estimator of the number of jumps in* $[0, T]$ *using our test, (which is formally a cumulative jump intensity estimator) and* $\Lambda^{actual}(T)$ *is the number of actually realized jumps in* $[0, T]$, *then the probability of global misclassification is*

$$P(\widehat{\Lambda} \neq \Lambda^{actual}|N\ Jumps) = P(GFTD\ or\ GSD|N\ jumps) = \frac{2}{\sqrt{2\pi}} y_n N + exp(-\beta_n) + o(exp(-\beta_n)).$$

It can be minimized at $\beta_n^* = -\log\left(\frac{\sigma\sqrt{T}N}{\sqrt{2n\log n}}\right)$. *Moreover, the overall optimal convergence rate is* $\frac{\sigma\sqrt{T}N}{\sqrt{2n\log n}}$.

Proof of Theorem 5: *See Appendix 6.7*

3 Monte Carlo Simulation

In this subsection, we study the effectiveness of the test through Monte Carlo simulation. Our asymptotic argument in previous sections requires that the sampling interval Δt converge to 0. This idealistic requirement cannot be perfectly met in real applications. This subsection investigates the finite sample performance of this test. The main result from this simulation shows that, as we increase the frequency of observation, the precision of our test increases. For the series generation, we used the Euler-Maruyama Stochastic Differential Equation (SDE) discretization scheme (Kloeden and Platen (1992)), an explicit order 0.5 strong and order 1.0 weak scheme. We discard the burn-in period - the first part of the whole series - to avoid the starting value effect. Throughout, we use the notation $\Delta t = \frac{1}{252 \times nobs}$, with $nobs$ as the number of observations per day.

3.1 Constant Volatility

We first consider the simplest model in the class with a fixed volatility. Table I presents the probability of SD_i. We simulate two constant volatility diffusion processes with fixed spot volatilities at realistic annualized values of 30% and 60%, respectively. A thousand series of returns over one year are simulated at several different frequencies from 1 to 288 observations per day - up to 5-minute returns. The significance level for this study is 5%. Table I shows that increasing the frequency of observation reduces the probability of SD_i.

Table II lists the probability of success to detect actual jumps, that is, one minus the probability of FTD_i. A thousand simulated tests at different frequencies, from one to 96 observations per day, are performed. Arrivals of six different jump sizes are assumed at 300% to 10% of the given volatility level of 30%. We chose different jump sizes to show that it is harder to detect smaller-sized jumps at low frequency. However, we show that as we increase the frequency, we

nobs	$\sigma = 0.3$	(SE)	$\sigma = 0.6$	(SE)	SV	(SE)
1	1.3305e-03	(7.4050e-05)	1.3305e-03	(7.6239e-05)	3.9110e-03	(1.3319e-04)
2	5.7380e-04	(3.4901e-05)	5.3222e-04	(3.3570e-05)	2.3306e-03	(7.7336e-05)
4	2.0696e-04	(1.4460e-05)	2.1209e-04	(1.4790e-05)	1.3289e-03	(4.3670e-05)
12	5.2879e-05	(4.3701e-06)	5.5911e-05	(4.2684e-06)	4.8131e-04	(1.6731e-05)
24	2.1775e-05	(1.9032e-06)	2.5126e-05	(2.0353e-06)	2.7688e-04	(1.0062e-05)
48	8.8436e-06	(8.3749e-07)	8.6768e-06	(8.3965e-07)	1.4467e-04	(7.2952e-06)
96	3.4947e-06	(3.7430e-07)	4.1876e-06	(4.2736e-07)	8.9449e-05	(4.2818e-06)
288	9.6810e-07	(1.1824e-07)	9.8630e-07	(2.2875e-07)	1.2986e-05	(1.2463e-06)

Table I: The mean and standard error (SE) of $P(SD_i)$, the probability of rejecting a spurious jump. The significance level α is 5%. The null model is a diffusion process with fixed volatilities, σ at 30% and 60% or with stochastic volatility (SV). *nobs* denotes the number of observations per day.

obtain very high detecting power (above 98%), even for very small sized jumps. For instance, from Table II, we can see that when the relative magnitude of jumps are 10% of volatility, econometricians are less likely to tell the difference between price changes due to the volatility part and those due to the jump part with lower frequency, such as daily. According to our study, they can detect the presence of jumps only 2% of the times using daily return. On the other hand, at frequencies as high as 30 minutes, we can distinguish the difference more than 95% of the times.

3.2 Stochastic Volatility

We examine how the test performs differently for stochastic volatility. Following the empirical study on realized variance of foreign currency exchange rates in Barndorff-Nielsen and Shephard (2004), we assume the spot volatility to be a sum of two uncorrelated square root processes [Cox, Ingersoll and Ross (1985)]. Specifically, the spot volatility process is modeled as a sum of separate

	Constant Volatility σ at 30%					
Jump Size	3σ	2σ	1σ	0.5σ	0.25σ	0.1σ
$nobs = 1$	0.9920	0.9880	0.9810	0.9270	0.4690	0.0260
	(0.0028)	(0.0034)	(0.0043)	(0.0082)	(0.0158)	(0.0050)
$nobs = 4$	0.9860	0.9780	0.9820	0.9700	0.9050	0.1520
	(0.0037)	(0.0046)	(0.0042)	(0.0054)	(0.0093)	(0.0114)
$nobs = 24$	0.9950	0.9860	0.9890	0.9890	0.9770	0.8880
	(0.0022)	(0.0037)	(0.0033)	(0.0033)	(0.0047)	(0.0100)
$nobs = 96$	0.9980	0.9970	0.9960	0.9920	0.9970	0.9820
	(0.0014)	(0.0017)	(0.0020)	(0.0028)	(0.0017)	(0.0042)
	Stochastic Volatility					
Jump Size	$3\widetilde{\sigma(t)}$	$2\widetilde{\sigma(t)}$	$1\widetilde{\sigma(t)}$	$0.5\widetilde{\sigma(t)}$	$0.25\widetilde{\sigma(t)}$	$0.1\widetilde{\sigma(t)}$
$nobs = 1$	0.9470	0.9330	0.8540	0.5720	0.2500	0.0320
	(0.0071)	(0.0079)	(0.0112)	(0.0157)	(0.0137)	(0.0056)
$nobs = 4$	0.9770	0.9690	0.9410	0.8480	0.5320	0.1400
	(0.0047)	(0.0055)	(0.0075)	(0.0114)	(0.0158)	(0.0110)
$nobs = 24$	0.9870	0.9860	0.9830	0.9610	0.8770	0.5260
	(0.0036)	(0.0037)	(0.0041)	(0.0061)	(0.0104)	(0.0158)
$nobs = 96$	0.9970	0.9990	0.9980	0.9920	0.9610	0.8100
	(0.0017)	(0.0010)	(0.0014)	(0.0028)	(0.0061)	(0.0130)

Table II: The mean and standard error (in parentheses) of $[1 - P(FTD_i)]$. The significance level α is 5%. The null model is a diffusion process with a fixed volatility σ at 30% and stochastic volatility. The jump sizes are determined compared to volatility. For stochastic volatility, the jump size depends on the mean of volatility $\widetilde{\sigma(t)} = E[\sigma(t)]$. $nobs$ denotes the number of observations per day.

solutions of two different stochastic differential equations,

$$d\sigma_s^2(t) = -\theta_s\{\sigma_s^2(t) - \kappa_s\}dt + \omega_s\sigma_s(t)dB(\theta_s t),$$

where B denotes a Brownian Motion, $\theta_s > 0$, and $\kappa_s \geq \omega_s^2/2$ for $s = 1, 2$. For this simulation, we use estimates calibrated by Barndorff-Nielsen and Shephard (2002) with exchange rate data in order for our study to mimic the real markets. The values are from

$$E(\sigma_s^2) = p_s 0.509, Var(\sigma_s^2) = p_s 0.461, \text{ for } s = 1 \text{and} 2,$$

with $p_1 = 0.218, p_2 = 0.782, \theta_1 = 0.0429,$ and $\theta_2 = 3.74$. We assume no correlation between two Brownian motions in volatility and the random terms in return process, which leaves us with no leverage effect in this simulation study. The jump size and the Poisson jump counting process are set to be the same as in the case of constant volatility. We include the result when the volatility is stochastic in Table I in order to allow direct comparison with the constant volatility case. It confirms our intuition that if the volatility moves over time, it would be more difficult to disentangle jumps. At every frequency, the corresponding success probability for stochastic volatility is greater than that with fixed volatility. We find the same result from the comparison in Table II: the probability of success in detecting a jump at some given time decreases under stochastic volatility. This shows that stochastic volatility reduces the precision of jump detection. However, this study does not alter our conclusion in the previous subsection; namely, if we increase the frequency of observation, we can still improve our ability to detect jumps. Indeed, for the case of $nobs = 96$, the success rate for stochastic volatility is as high as for constant volatility.

3.3 Global Misclassification

In this subsection, we examine the global likelihood of misclassification by either *GSD* or *GFTD*, as studied in Theorem 5. This tells us how accurately we can locate actual jump arrival times.

We simulate five-hundred different series of one year observations at different frequencies from one to 96 observations per day - from daily to 15-minute returns. We consider five different jump sizes from 3 times to 25% of volatility level. As discussed in Theorem 5, we find the likelihood becomes negligible at higher frequencies. Table III presents the probability that the number of jumps counted by our test is not equal to the actual number of jumps.

3.4 Comparison with Other Jump Tests

The comparable tests to ours are those introduced by Barndorff-Nielsen and Shephard (2006) (BNS) and Jiang and Oomen (2005) (JO), both of which are also nonparametric, making test results robust to model specification. This subsection explains the difference between these tests and ours. BNS takes the difference (or ratio) between the realized quadratic variation and bipower variation during a certain time interval to distinguish the presence of jumps in that interval. JO's swap variance test takes a similar approach to BNS's. Instead of using bipower variation, JO uses cumulative delta-hedged gain or loss of a variance swap replicating strategy. They both also provide asymptotic null distributions for their jump test statistics. As stated in BNS, after simulation study under alternative hypotheses, one of the features of their test is that it cannot distinguish two jumps a day with low variance and one jump with high variance in terms of rejection rates. The reason for this problem is that their test depends on integrated quantities. JO's swap variance test shares this feature because it also depends on an integrated quantity. The following example illustrates the difference more clearly. Suppose there are two jumps in a day and an analyst chooses one day as the interval for their test. The presence of a jump in that day can be recognized, but not how many jumps occurred, whether the jump(s) was negative or positive, and at what time of the day the jump(s) occurred. These issues can, however, be resolved by our test.

Probability of Global Misclassification					
Jump Size	3σ	2σ	1σ	0.5σ	0.25σ
$nobs = 1$	0.2140	0.2120	0.1640	0.3540	0.9860
	(0.0184)	(0.0183)	(0.0166)	(0.0214)	(0.0053)
$nobs = 2$	0.1252	0.1312	0.1531	0.1173	0.7714
	(0.0148)	(0.0151)	(0.0161)	(0.0144)	(0.0188)
$nobs = 4$	0.0477	0.0407	0.0407	0.0506	0.0755
	(0.0095)	(0.0088)	(0.0088)	(0.0098)	(0.0118)
$nobs = 12$	0.0036	0.0046	0.0023	0.0063	0.0050
	(0.0027)	(0.0030)	(0.0021)	(0.0035)	(0.0031)
$nobs = 24$	0.0012	0.0012	0.0005	0.0007	0.0008
	(0.0015)	(0.0015)	(0.0010)	(0.0011)	(0.0013)
$nobs = 48$	0.0001	0.0001	0.0006	0.0004	0.0002
	(0.0003)	(0.0004)	(0.0011)	(0.0009)	(0.0006)
$nobs = 96$	0.0001	0.0001	0.0001	0.0002	0.0000
	(0.0001)	(0.0002)	(0.0005)	(0.0006)	(0.0001)

Table III: The mean and standard error (in parentheses) of probability of global misclassification, $P\left(\Lambda(T) \neq \Lambda^{actual}(T)\right)$. The significance level α is 5%. The null model is a diffusion process with stochastic volatility.

Not only can our test do more analysis, as explained above, but it also outperforms the BNS test on the same analysis. We only do a comparative study with BNS, since, except for the convergence rate to the asymptotic distribution, the same results are expected from JO. In the rest of this subsection, we report a simulation study comparing the power of our new test to BNS's linear test. We choose their linear test because it performs better than their adjusted ratio test in their simulation study, despite the little difference. We therefore presume that demonstrating our test outperforms their linear test is sufficient.

We design this simulation by introducing one or two jumps a day to a diffusion process with a constant volatility. We consider 3000 simulated series of a process in a day, with each jump arriving randomly as a Poisson process. The number of jumps was set to be either one or two. If there is a given number of jumps for some time, (one day in this study), then the Poisson jump arrival time is uniformly distributed [see Ross (1995)]. Hence, we randomly select the arrival times from a uniform distribution. σ is set at 30%, as in the previous simulation. The variance of jump size distribution is chosen at 10%, 5%, 1% and 0.5% of σ^2. At higher variances of jump sizes, the difference between the two tests is not large. Table IV shows the cases with lower variances, which demonstrate better performance of our test. In conclusion, our test performs equally or better for all jump sizes, numbers of jumps, and frequencies.

3.5 Optimal Window Size K

The optimal choice for window size is studied by simulation in this subsection. In the plots, we show the relationship between mean squared error and the choice of window size. Four cases are plotted in Figure 2 to show the optimal window size. The upper left panel shows mean squared error as a function of window size K, using daily data with the horizon $T = 1$ year. The remaining

	Number of Jumps per Day $N = 1$				Number of Jumps per Day $N = 2$			
10%	Linear Test (BNS)		Our Test		Linear Test (BNS)		Our Test	
nobs	$1 - FTD$	(SE)	$1 - FTD$	(SE)	$1 - FTD$	(SE)	$1 - FTD$	(SE)
48	0.8520	(0.0112)	0.8717	(0.0107)	0.9690	(0.0055)	0.9820	(0.0042)
72	0.8819	(0.0102)	0.8967	(0.0097)	0.9860	(0.0037)	0.9879	(0.0036)
96	0.8987	(0.0091)	0.9220	(0.0085)	0.9860	(0.0037)	0.9890	(0.0033)
288	0.9360	(0.0075)	0.9620	(0.0060)	0.9930	(0.0026)	0.9990	(0.0010)
5%	Linear Test (BNS)		Our Test		Linear Test (BNS)		Our Test	
nobs	$1 - FTD$	(SE)	$1 - FTD$	(SE)	$1 - FTD$	(SE)	$1 - FTD$	(SE)
48	0.8040	(0.0123)	0.8480	(0.0114)	0.9570	(0.0064)	0.9730	(0.0051)
72	0.8248	(0.0120)	0.8509	(0.0113)	0.9690	(0.0055)	0.9770	(0.0047)
96	0.8347	(0.0117)	0.8740	(0.0105)	0.9720	(0.0052)	0.9810	(0.0043)
288	0.8900	(0.0096)	0.9340	(0.0079)	0.9890	(0.0033)	0.9960	(0.0020)
1%	Linear Test (BNS)		Our Test		Linear Test (BNS)		Our Test	
nobs	$1 - FTD$	(SE)	$1 - FTD$	(SE)	$1 - FTD$	(SE)	$1 - FTD$	(SE)
48	0.5650	(0.0157)	0.6310	(0.0153)	0.8160	(0.0123)	0.8670	(0.0107)
72	0.6640	(0.0149)	0.7130	(0.0140)	0.8677	(0.0107)	0.9039	(0.0093)
96	0.6470	(0.0151)	0.7330	(0.0143)	0.8770	(0.0104)	0.9260	(0.0083)
288	0.7610	(0.0135)	0.8470	(0.0114)	0.9490	(0.0070)	0.9740	(0.0050)
0.5%	Linear Test (BNS)		Our Test		Linear Test (BNS)		Our Test	
nobs	$1 - FTD$	(SE)	$1 - FTD$	(SE)	$1 - FTD$	(SE)	$1 - FTD$	(SE)
48	0.4603	(0.0158)	0.5375	(0.0158)	0.6857	(0.0147)	0.7668	(0.0134)
72	0.4940	(0.0158)	0.6127	(0.0155)	0.7570	(0.0136)	0.8230	(0.0121)
96	0.5493	(0.0157)	0.6590	(0.0150)	0.8070	(0.0125)	0.8740	(0.0105)
288	0.6753	(0.0145)	0.7990	(0.0127)	0.9130	(0.0089)	0.9570	(0.0064)

Table IV: The probabilities and standard errors (in parenthesis) of rejecting the null hypothesis of no jump with the linear test by Barndorff-Nielsen and Shephard (2006) (BNS) and our new test, given one or two jumps a day. The variance of jumps are 10%, 5%, 1% and 0.5% of σ^2.

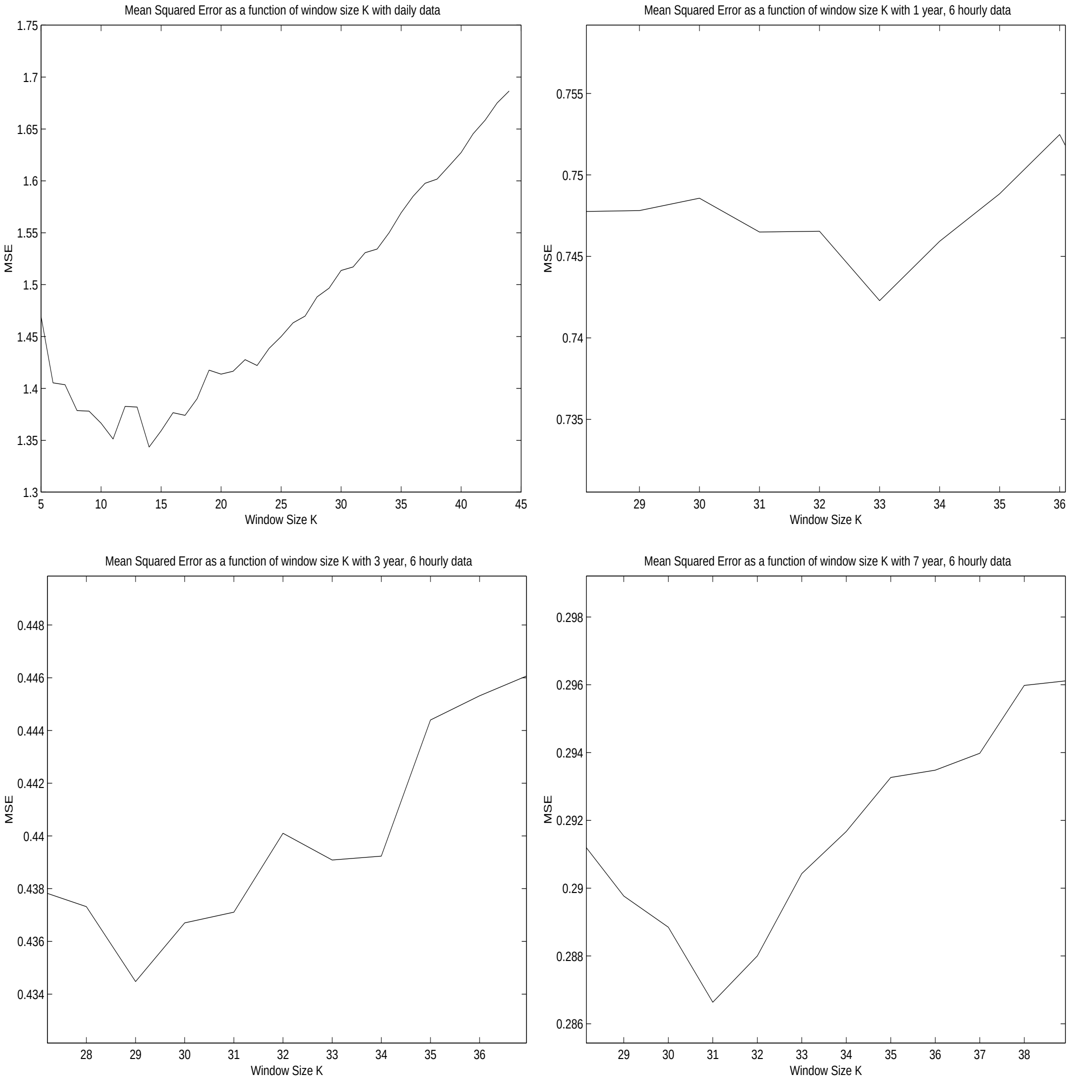

Figure 2: Graphs of mean squared total error with daily and 6-hourly returns, as a function of window size K.

three use 6-hourly return data with the horizons $T = 1, 3$, and 7 years. It turns out that it is optimal for K to be the smallest integer in the condition set, $K = O_p(\Delta t^{\alpha})$ with $-1 < \alpha < -0.5$, because it gives the lowest mean squared error in our simulation. Increasing window size would not always increase the test's efficiency, especially for the intensity estimation, because increasing window size lowers the number of multiple tests for a fixed number of observations.

4 Applications: Dynamics of Jump Arrivals

One of the advantages of our new test is that it facilitates study of jump dynamics up to at intra-day levels. Not only can it lead to improved models for many empirical studies that involve jumps, but also allows investigation of unknown phenomenon in real-time markets.

4.1 Application to Option Pricing Models

Improvements in continuous-time option pricing models with time-varying jump intensity have already been stressed in a number of theoretical and empirical studies [see, for example, Bates (2000, 2002), Duffie, *et al* (2000), Andersen, Benzoni, and Lund (2002), Pan (2002), Chernov *et al* (2003), Dubinsky and Johannes (2005)]. In these studies, volatility levels, jump sizes, or earnings announcements were chosen to explain time-varying jump intensities in linear and non-linear fashions. As reported earlier in this paper, only a few (2%) obvious jumps can be detected and estimated using daily observations, which all of aforementioned empirical studies employed. Using our more advanced econometric technique, in-depth investigation on jump dynamics using a high-frequency time series of underlying securities on which options depend is first warranted. Following this, cross-sectional option data can be used to pin down the market price of jump size or intensity risks based on the predetermined model.

4.2 Empirical Analysis

In this paper, we conduct an empirical study to explore real-time jump dynamics in individual U.S. stock prices. From the Trade and Quote (TAQ) database, we collected ultra-high-frequency stock prices and generate returns by taking differences of log prices. We multiply all returns by 100 to present them as percentages. The time span used is the 3 months from September 1 to November 30, 2005, which was the most recent data available, and never previously investigated. We chose four major U.S. companies: Wal-Mart (WMT), IBM (IBM), Coca Cola (KO), and General Electric (GE). Using stock prices from the New York Stock Exchanage (NYSE), 15-minute returns were used for the results in Table V. As proven in the simulation in Section 3, as 15-minute return is enough to achieve a decent power for the test. With this choice of frequency, our empirical results are not greatly affected by the presence of market microstructure noise. Therefore, we ignore the effect of the noise in this particular study. The significance level for all series is 5%. The outcomes of the tests are each arrival date, time, jump size, and mean and variance of the detected jump size distribution. We do not assume that there is no more than one jump per day, but we do assume that when a jump occurs, the jump size dominates the return. We find that most of the jumps in equity prices under consideration arrive around the time of market opening. Except for one or two jumps, they were associated with news events. Using Factiva, we searched for real-time business news and information releases around jump arrival times. Sources for Factiva include the Wall Street Journal, the Financial Times, and Dow Jones and Reuters newswires. During 3 months, there was one scheduled corporate news event for each company, which was the third-quarter earnings announcement. Around earnings announcement times, we find similar evidence as Dubinsky and Johannes (2005), who set an equity option pricing model with jump events conditional on deterministic earnings announcement date (EAD). In addition to prescheduled announcements, however, we find the evidence of stock price jumps' association with more of unscheduled news events. We also discover that the sizes of jumps associated with earnings announcements are not necessarily the largest in our sample, which suggests strongly that scheduled events are not sufficient for an individual equity pricing model with jumps. Deepening our understanding on jump dynamics of individual equities can also lead to exploiting a profitable trading strategy.

5 Concluding Remarks

We introduce a new nonparametric test to detect realized jumps up to at intra-day frequency and characterize their sizes. We show that jump dynamics and overall return distributions of any type of security can be investigated. Monte Carlo simulation experiments show that intra-day time series data increase the precision of jump tests, decreasing the likelihood of global misclassification of jumps. Our empirical study gives strong evidence of an association between jump arrivals and both prescheduled earnings announcements and real-time news releases in U.S. equity markets. Our test's results are robust to model specification and nonstationarity of the series applied. We consider the Poisson type of jumps in pricing models in this paper. One possible future direction for research is to develop a test for jumps from pure jump processes, such as infinite activity Levy processes. Another extension is to consider the presence of market microstructure noise at ultra-high frequencies of data, which we are currently developing.

Wal Mart (WMT)					IBM (IBM)				
Date	Time	Size (%)	News	S	Date	Time	Size (%)	News	S
Sep 26	9:30am	1.29	Law Suit	N	Sep 13	9:45am	-0.76	Announcement	N
Oct 06	9:31am	1.12	EAD	Y	Sep 19	9:30am	-0.74	Deal News	N
Oct 10	9:30am	1.53	New Launch	N	Sep 21	9:30am	-0.96	Option Market	N
Oct 14	9:30am	0.93	Law Suit	N	Sep 27	9:45am	1.03	Good News	N
Oct 19	10:45am	0.84	Investment	N	Oct 07	9:45am	1.04	Good News	N
Oct 31	9:30am	1.33	Sales Up	N	Oct 10	9:30am	0.98	New Product	N
Nov 15	9:48am	-1.25	Announcement	N	Oct 11	9:30am	1.24	Expansion	N
Nov 18	9:30am	1.11	Announcement	N	Oct 18	9:30am	1.99	EAD	Y
					Oct 19	9:30am	-1.29	Rival EAD	Y
					Nov 18	9:30am	1.30	Announcement	N
$\mu_y = 0.8625$		$\sigma_y = 0.8817$			$\mu_y = 0.3830$		$\sigma_y = 1.1801$		
General Electric (GE)					Coca Cola (KO)				
Date	Time	Size (%)	News	S	Date	Time	Size (%)	News	S
Sep 15	9:45am	0.70	Good News	N	Sep 13	9:45am	-1.59	Bad News	N
Sep 21	9:30am	-0.95	Bad News	N	Sep 21	9:30am	0.96	Announcement	Y
Oct 06	9:31am	2.03	Announcement	N	Oct 20	9:30am	2.60	EAD	Y
Oct 07	9:30am	0.92	Good News	N	Nov 18	9:30am	1.42	Announcement	N
Oct 14	9:30am	1.11	EAD	Y	Nov 18	9:45am	-0.83	Announcement	N
Nov 18	9:30am	2.20	Announcement	N	Nov 29	9:30am	-0.88	Announcement	N
$\mu_y = 1.0017$		$\sigma_y = 1.1324$			$\mu_y = 0.28$		$\sigma_y = 1.6260$		

Table V: Jump dates, jump times, and observed jump sizes, with associated news events. μ_y and σ_y are mean and standard deviation of observed jump sizes. The result is based on transaction prices from the NYSE during 3 months from September 1 to November 30, 2005. The significance level of each test is set at 5%. S denotes whether the news was scheduled or not. EAD denotes an earnings announcement date

6 Appendix

6.1 The Non-zero Drift

The main conclusion of Theorem 1 is not altered under an extension to the non-zero drift case. Suppose now, we have the non-zero drift coefficient $\mu(t)$ for the return process; that is,

$$d \log S(t) = \mu(t)dt + \sigma dW(t).$$

A modified version of Definition 1 for this case is as follows:

Definition 1.1 *The statistic $T_\mu(t_i)$, which tests at time t_i whether there was a jump from t_{i-1} to t_i, is defined as*

$$T_\mu(t_i) = \frac{\log S(t_i) - \log S(t_{i-1}) - \widehat{m}_i}{\sqrt{\frac{1}{K-2}\sum_{j=i-K+2}^{i-1} |\log S(t_j) - \log S(t_{j-1})||\log S(t_{j-1}) - \log S(t_{j-2})|}},$$

where

$$\widehat{m}_i = \frac{1}{K-1}\sum_{j=i-K+1}^{i-1} (\log S(t_j) - \log S(t_{j-1})).$$

Then, we can extend Theorem 1 to the case of non-zero drift as below.

Theorem 1.1 *Let $T_\mu(t_i)$ be as in* **Definition 1.1** *under the null and $K = O_p(\Delta t^\alpha)$, where $-1 < \alpha < -0.5$. Suppose* **Assumption 1** *is satisfied. Then, as $\Delta t \to 0$,*

$$sup_i |T_\mu(t_i) - \widehat{T}_\mu(t_i)| = O_p(\Delta t^{\frac{3}{2}-\delta+\alpha}),$$

where

$$\widehat{T}_\mu(t_i) = \frac{U_i - \bar{U}_{i-1}}{c}.$$

Here, $\bar{U}_{i-1} = \frac{1}{K-1}\sum_{j=i-K+1}^{i-1} U_j$ *and the rest of notations are the same as in* **Theorem 1**.

Proof of Theorem 1.1: *See Appendix 5.2.*

The error rate is the same as the zero drift case, since the error due to the drift term is dominated by the error due to the diffusion part. $\widehat{T}_{\mu}(t_i)$ asymptotically follows a normal distribution with its mean 0 and variance $\frac{K}{c^2(K-1)} \to \frac{1}{c^2}$, since $K \to \infty$. Our choice of K makes the effect of jumps on $\hat{m}_i$ vanish because of the property of the Poisson process for rare jumps, which says that there can be no more than a single jump in an infinitesimal time interval. Because this makes only a finite number, say L, of jumps in the window, the jump test statistic for the non-zero drift case becomes

$$T(t_i) \approx \frac{U_i - \bar{U}_{i-1}}{c} - \frac{L \times Y(\tau)}{c\sigma(K-1)\sqrt{\Delta t}} I_{\tau \in (t_{i-K}, t_{i-1}]}.$$

The second term will disappear because of the condition $K\sqrt{\Delta t} \to \infty$. This shows that jumps in the window have asymptotically negligible effect on testing jumps from t_{i-1} to t_i with our choice of K.

6.2 Proof of Theorem 1 and Theorem 1.1 in Appendix 6.1

For $t_{i-K} < t < t_i$,

$$\log S(t) - \log S(t_{i-K}) = \int_{t_{i-K}}^{t} \mu(u)du + \int_{t_{i-K}}^{t} \sigma(u)dW(u).$$

Given the Assumption 1 imposed, we have with A1.1,

$$\int_{t_{i-1}}^{t_i} \mu(u)du - \mu(t_{i-1})\Delta t = O_p(\Delta t^{\frac{3}{2}})$$

$$\text{and} \int_{t_{i-K}}^{t_i} \mu(u)du - \mu(t_{i-K})K\Delta t = O_p(\Delta t^{\frac{3}{2}+\alpha})$$

uniformly in all i. This implies

$$sup_{i,t\leq t_i}|\int_{t_{i-K}}^{t}\{\mu(u)-\mu(t_{i-K})\}du| = O_p(\Delta t^{\frac{3}{2}+\alpha}).$$

Similarly to *lemma* 1 in Mykland and Zhang (2001), under the condition A1.2, we can apply Burkholder's Inequality (Protter (1995)) to get

$$sup_{i,t\leq t_i}|\int_{t_{i-K}}^{t}\{\sigma(u)-\sigma(t_{i-K})\}dW(u)| = O_p(\Delta t^{\frac{3}{2}-\delta+\alpha}),$$

where δ can be any number in $0<\delta<\frac{3}{2}+\alpha$. This result is also uniform in i for $K = O_p(\Delta t^{\alpha})$ as specified. Therefore, over the window, for $t \in [t_{i-K}, t_i]$, $dlogS(t)$ can be approximated by $dlogS^i(t)$, such that

$$dlogS^i(t) = \mu(t_{i-K})dt + \sigma(t_{i-K})dW(t),$$

because

$$\begin{aligned}
&|\left(\log S(t)-\log S(t_{i-K})\right) - \Big(\log S^i(t) - \log S^i(t_{i-K})\Big)| \\
&= |\int_{t_{i-K}}^{t}(\mu(u)-\mu(t_{i-K}))du + \int_{t_{i-K}}^{t}(\sigma(u)-\sigma(t_{i-K}))dW(u)| = O_p(\Delta t^{\frac{3}{2}-\delta+\alpha}).
\end{aligned}$$

For all i, j and $t_j \in [t_{i-K}, t_i]$, the numerator is

$$\begin{aligned}
&\log S(t_j) - \log S(t_{j-1}) - \widehat{m}_i \\
&= \log S^i(t_j) - \log S^i(t_{j-1}) - \frac{1}{K-1}\sum_{l=i-K+1}^{i-1}\Big(\log S^i(t_l) - \log S^i(t_{l-1})\Big) + O_p(\Delta t^{\frac{3}{2}-\delta+\alpha}) \\
&= \sigma(t_{i-K})W_{\Delta t} - \frac{1}{K-1}\sum_{l=i-K+1}^{i-1}\sigma(t_{i-K})W_{\Delta t} + O_p(\Delta t^{\frac{3}{2}-\delta+\alpha}) \\
&= \sigma(t_{i-K})\sqrt{\Delta t}(U_j - \bar{U}_{i-1}) + O_p(\Delta t^{\frac{3}{2}-\delta+\alpha}),
\end{aligned}$$

where $U_j = \frac{1}{\sqrt{\Delta t}}(W_{t_j} - W_{t_{j-1}}) \sim iid$ Normal$(0,1)$ and $\bar{U}_{i-1} = \frac{1}{K-1}\sum_{j=i-K+1}^{i-1} U_j$.

For the denominator, we put a volatility estimator based on the realized bipower variation for integrated volatility estimator [see Barndorff-Nielsen and Shephard (2004)]. According to *Proposition*

2 in Barndorff-Nielsen and Shephard (2004), the impact of the drift term is negligible; hence, it does not affect the asymptotic limit behavior. Then, we are left to prove the following approximation of scaled volatility estimator:

$$\text{plim}_{\Delta t \to 0} c^2 \hat{\sigma}^2(t) = \text{plim}_{\Delta t \to 0} \frac{1}{(K-2)\Delta t} \sum_j |\log S(t_j) - \log S(t_{j-1})||\log S(t_{j-1}) - \log S(t_{j-2})|$$
$$= c^2 \sigma^2(t).$$

It is due to

$$\frac{1}{(K-2)\Delta t} \sum_{j=i-K+3}^{i-1} |\log S(t_j) - \log S(t_{j-1})||\log S(t_{j-1}) - \log S(t_{j-2})|$$
$$= \frac{1}{(K-2)\Delta t} \sum_{j=i-K+3}^{i-1} |\log S^i(t_j) - \log S^i(t_{j-1}) + O_p(\Delta t^{\frac{3}{2}-\delta+\alpha})||\log S^i(t_{j-1}) - \log S^i(t_{j-2}) + O_p(\Delta t^{\frac{3}{2}-\delta+\alpha})|$$
$$= \frac{1}{(K-2)\Delta t} \sum_{j=i-K+3}^{i-1} |\log S^i(t_j) - \log S^i(t_{j-1})||\log S^i(t_{j-1}) - \log S^i(t_{j-2})| + O_p(\Delta t^{\frac{3}{2}-\delta+\alpha})$$
$$= \frac{1}{K-2} \sum_{j=i-K+3}^{i-1} \sigma^2(t_{i-K})|\sqrt{\Delta t}U_j||\sqrt{\Delta t}U_{j-1}| + O_p(\Delta t^{\frac{3}{2}-\delta+\alpha})$$
$$= \sigma^2(t_{i-K})c^2 + O_p(\Delta t^{\frac{3}{2}-\delta+\alpha}),$$

where U_i's are iid $Normal(0,1)$ and $c = E(|U_i|) \approx 0.7979$. Then,

$$T(t_i) = \frac{(U_i - \bar{U}_{i-1})}{c} + O_p(\Delta^{\frac{3}{2}-\delta+\alpha}).$$

This proves Theorem 1 and Theorem 1.1.

Alternatively, for the non-zero drift case, we can use Girsanov's Theorem to suppose $\mu(t) = 0$, as in Zhang, Mykland and Ait-Sahalia (2005).

6.3 Proof of Theorem 2

When there is a possibility of rare Poisson jumps in the window, the scaled bipower variation $c^2\hat{\sigma}^2(t)$ can be decomposed into two parts: one with jump terms and one without jump terms, as follows.

$$c^2\hat{\sigma}^2(t) = \frac{1}{(K-2)\Delta t}\sum_{j=i-K+2}^{i-1} |\log S(t_j) - \log S(t_{j-1})||\log S(t_{j-1}) - \log S(t_{j-2})|$$

$$= \text{terms without jumps} + \frac{1}{(K-2)\Delta t}\sum_{\text{terms with jumps}} \sigma(t_j)|\Delta W_{t_j}||Jump|$$

$$= \text{terms without jumps} + \frac{1}{(K-2)\Delta t}O_p(\sqrt{\Delta t})\sum_{\text{terms with jumps}} \sigma(t_j)|Jump|$$

$$= \text{terms without jumps} + \frac{1}{(K-2)\Delta t}O_p(\sqrt{\Delta t}).$$

The order of the second term is due to the property of the Poisson jump process that allows a finite number of jumps over the window. Since $\sigma(t_j)|Jump| = O_p(1)$, $\sum_{\text{terms with jumps}} \sigma(t_j)|Jump| = O_p(1)$. The effect of jump terms becoming negligible requires the second term to be $o_p(1)$ as Δt goes to 0. The window size K that satisfies $K\sqrt{\Delta t} \to \infty$ and $K\Delta t \to 0$ as Δt goes to 0 will work. If we assume the window size to be $K = \beta \Delta t^{\alpha}$, where β is some constant, then the necessary condition for α is $-1 < \alpha < -0.5$. Accordingly,

$$lim_{\Delta t\to 0}\hat{\sigma}^2|_{alternative} = lim_{\Delta t\to 0}\hat{\sigma}^2|_{null} = \sigma^2(t).$$

Then, putting the approximation for return above in the statistic yields

$$T(t_i) \approx \frac{\sigma_{t_{i-K}}\sqrt{\Delta t}U_i + Y(\tau)I_{\tau\in(t_{i-1},t_i)}}{c\sigma(t_{i-K})\sqrt{\Delta t}} = \frac{U_i}{c} + \frac{Y(\tau)}{c\sigma\sqrt{\Delta t}}I_{\tau\in(t_{i-1},t_i)}.$$

6.4 Proof of Lemma 1

Proof of Lemma 1 follows from Aldous (1985) and the proof in Galambos (1978).

6.5 Proof of Theorem 3

We suppose there are N jumps from time $t = 0$ to $t = T$ and claim there is a jump if $|T(t_i)| > \beta_n S_n + C_n$. Fix a set of jump times as $A_n = \{i : \text{there is a jump in } (t_{i-1}, t_i]\}$. Then,

$$P(\text{We correctly classify all N jumps}|\text{N jumps}) = P(\text{For all } i \in A_n, |T(t_i)| > \beta_n S_n + C_n)$$

$$\approx \prod_{i\in A_n} P(|T(t_i)| > \beta_n S_n + C_n) \approx \prod_{i\in A_n} P(|Y(t_i)| > (\beta_n S_n + C_n) c\sigma\sqrt{\Delta t})$$

$$= \prod_{i\in A_n} \Big(1 - F_{|Y|}(y_n)\Big) \sim \Big(1 - \frac{2}{\sqrt{2\pi}} y_n + o(y_n^2)\Big)^N = 1 - \frac{2}{\sqrt{2\pi}} y_n N + o(y_n^2 N).$$

6.6 Proof of Theorem 4

Let $A_n^C = \{0, 1, ..., n-1\} - A_n$ be a set of non-jump times. Then,

$$P(\text{We incorrectly reject any non-jumps}|\text{N jumps})$$

$$= P(\text{for some } i \in A_n^C, |T(t_i)| > \beta_n S_n + C_n|\text{N jumps})$$

$$= P(max_{i\in A_n^C}|T(t_i)| > \beta_n S_n + C_n|\text{N jumps})$$

$$\approx P(max_{i\in A_n^C}|\widehat{T}(t_i)| > \beta_n S_n + C_n) = 1 - F_\xi(\beta_n) = exp(-\beta_n) + o(exp(-\beta_n)).$$

By L'Hopital's rule, we obtain the last step because

$$lim_{\beta_n\to\infty} \frac{1 - F_\xi(\beta_n)}{exp(-\beta_n)} = 1.$$

6.7 Proof of Theorem 5

Prob(*GMJ or GMNJ*)=Prob(*GMJ*)+Prob(*GMNJ*) and the results follows from Theorems 3 and 4. Minimum probability can be achieved at β_n^*, which can be obtained by taking the first derivative of probability with respect to β_n and setting it equal to 0, as

$$\frac{\partial P}{\partial \beta_n} = \frac{2}{\sqrt{2\pi}} S_n \sigma c\sqrt{\Delta t} N - exp(-\beta_n) = 0.$$

7 References

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