In machine learning society, there are many works using neural networks solving partial differential equations (PDEs). Most works can be summarized into two categories: (1) formulate the problems into a least-squares minimization problems and using gradient-based optimization method to search for the minimizer. (2) use neural networks to parametrize trial functions and minimize the corresponding variational form. Both methods show promises but lack theoretical guarantees.

In this paper, we propose an optimization method which utilizes the convexity of variational form. We show that gradient descent in function space in terms of a neural-network provably finds the minimum of variational form as long as the step size is controlled. Simulations showed that our algorithm is comparable to Physical-informed neural networks (PINNs) and DeepRitz, which are two neural-network-based PDEs solvers.

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