THE UNIVERSITY OF CHICAGO

Department of Statistics

MASTER'S THESIS PRESENTATION

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Statistics of the Shepp p-Product

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ZOOM Meeting

ABSTRACT

In 1962, Shepp famously discovered a product of normal random variables that preserves normality. The Shepp product, which takes the form XY/(X^2 + Y^2)^1/2, has since been thoroughly studied and has found numerous connections to other areas of statistics. Among other things, it has an extension to n normal variables, gives a multiplicative analogue of central limit theorem, and applies unexpectedly to genomics as a test statistics for alignment-free sequence analysis. The Shepp product is evidently the p = 2 special case of XY/(X^p + Y^p)^1/p that we call the Shepp p-product. We will show that the Shepp p-product, particularly when p = 1 and ∞ (the latter in a limiting sense), is no less fascinating and applicable than the original p = 2 case. Just as the Shepp 2-product preserves normal distributions, the Shepp 1-product preserves Cauchy distributions while the Shepp ∞-product preserves exponential distributions. In fact, the converse is also true in an appropriate sense, allowing us to characterize the Cauchy, normal, and exponential distributions as the unique distributions preserved by the Shepp p-product for p = 1, 2, ∞ respectively. We will study the multiplicative analogue of infinite divisibility with respect to the Shepp p-product, establish an asymptotic theory for the Shepp p-product of n i.i.d. random variables, and estimate the rates of convergence in Kolmogorov distance. Alongside our study of convergence rates, we define the domain of normal attraction of extremal distributions and establish a new rate of uniform convergence to Frechet distribution and reverse Weibull distribution. Some of our results are new even for the p = 2 case. We will also discuss new applications of the Shepp p-product in statistics, computational biology, and statistical physics.
