ABSTRACT

We will discuss three problems in probability theory with close relations to percolation theory and critical phenomena in statistical physics: the loop isomorphism theorem, the coupling between the Brownian and the random walk loop soup occupation fields, and the shape theorem for the Poisson cylinder model. We first explore the (discrete loop) isomorphism theorem, which connects discrete Gaussian free fields (GFFs) squared and discrete loop soup occupation fields with the same underlying infinitesimal generator. This theorem has been used to study percolation properties of the level sets of the GFFs and cluster properties of the loop soup model. We generalize the existing isomorphism theorem and provide a simpler proof. We then couple the planar Brownian and the random walk loop soup occupation fields. We give an invariance principle that generalizes the continuous version of the loop isomorphism theorem. This invariance principle should be useful in studying the Brownian loop soup occupation field by approximating it with the random walk analogue, and vice versa. Finally, we discuss a Poisson cylinder model, which is an almost surely connected random set of unit-radius bi-infinite cylinders sampled uniformly at random in a Euclidean space. We establish a bound on the internal distances induced by this cylinder set and use it to prove a shape theorem for Poisson cylinders.