THE UNIVERSITY OF CHICAGO | DEPARTMENT OF STATISTICS

Statistics Colloquium

Joint colloquium with the
Committee on Computational and Applied Mathematics (CCAM)

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Brandeis University

"Fractal Geometry of Stochastic Partial Differential Equations"

Tuesday, January 30, 2024, at 4:00 PM
Jones 303, 5747 S. Ellis Avenue
Refreshments will be served prior to the Seminar at 3:30 pm in Jones 303

Abstract

Stochastic partial differential equations (PDEs) find extensive applications across diverse domains such as physics, finance, biology, and engineering, serving as effective tools for modeling systems influenced by random factors. The analysis of the patterns in the peaks and valleys of stochastic PDEs is crucial for gaining deeper insights into the underlying physical phenomena.

One notable example is the KPZ equation, a fundamental stochastic PDE associated with significant models like random growth processes, Burgers turbulence, interacting particle systems, and random polymers. The study of the fractal structures inherent in the KPZ equation provides a quantitative characterization of the intermittent nature of its peaks, as well as those of the stochastic heat equation—a subject that has been extensively explored over the past few decades.

Conversely, the Parabolic Anderson model (PAM) serves as a prototypical framework for simulating the conduction of electrons in crystals containing defects. Investigating the intermittency of peaks in the PAM has been a prominent area of research, closely tied to the phenomenon of Anderson localization.

In this presentation, we delve into the fractal geometry of both the KPZ equation and the PAM, unveiling their multifractal nature. Specifically, we demonstrate that the spatial and spatio-temporal peaks of these equations exhibit infinitely many distinct values. Furthermore, we compute the macroscopic Hausdorff dimension (introduced by Barlow and Taylor) associated with these peaks.

The key findings presented here stem from a series of works that employ a diverse array of tools, ranging from random matrix theory and the Gibbs property of random curves to the utilization of regularity structures and paracontrolled calculus. To conclude, we touch upon some open avenues for further research in this dynamic field.

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