“Logistic regression with small Bayes error”

MONDAY, MAY 1, 2023 at 4:30 PM
Jones 303, 5747 S. Ellis Avenue
Refreshments before the seminar at 4:00 PM in Jones 303.

Abstract

It is well-known that in binary classification the logistic regression estimator is unstable when the Bayes error $\sigma$ is small. Yet, a small Bayes error should in fact be helpful and result in an improved rate for estimating the direction of the regression coefficients. We show that this is indeed the case, at least in a low-dimensional situation where the number of variables $d$ is smaller than the number of observations $n$. If $\sigma$ is small, but not too small ($\sigma \gtrsim d \log n/n$) we obtain the rate $\sqrt{\sigma d \log n/n}$. If $\sigma$ is very small ($\sigma \lesssim d \log n/n$) the logistic regression estimator might not exist as there might be perfect separation. A ridge penalty on the regression coefficients however prevents interpolation and one finds the rate $\sqrt{\sigma d \log n/n} \asymp d \log n/n$ also in this very low noise case. Up to the log-term, this rate coincides with the one for interpolation in the noiseless case. The results rely on the assumption of Gaussian design and additive sub-Gaussian noise.