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DEPARTMENT OF STATISTICS

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“Robust and Range-Based GARCH Models with Leverage Effects: A Study of VaR–ES Forecasting”

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Abstract

While the non-asymptotic volatility modeling plays a key role in managing financial tail risk. This thesis explores approaches that integrate range-based information, robustness, and leverage asymmetry within GARCH frameworks. Specifically, it extends bounded M-estimation and range estimation to leverage settings, introducing two models: the BM–GJR–GARCH and a new leverage-based robust range model, the BM–GJR–RGARCH. Both use empirically calibrated robustness cutoffs to improve estimation stability. Using simulations and daily OHLC data for AAPL, TSLA, SPY, IEF, AGG, HYG, and USO from 2010 to 2024, the models are evaluated through Value-at-Risk (VaR) and Expected Shortfall (ES) forecasts at 1% and 5% confidence levels. Performance is assessed using coverage, duration, and joint VaR–ES loss tests. Results show that robust leverage models, particularly the Student-t BM–GJR–RGARCH, achieve stronger tail performance for equities, though optimal specifications vary across assets and tail levels. Overall, the findings suggest that combining leverage effects, range-based measures, and data-driven robustness can substantially enhance the reliability of VaR and ES forecasts under stress conditions.

Asymptotic convergence of the stochastic gradient descent (SGD) has been extensively studied, most existing works focus on convergence in terms of the mean squared error (MSE). The behavior of SGD with respect to higher moments remains under explored. This paper addresses this gap by establishing L_p convergence of SGD for any $p \geq 2$. We consider both constant learning rates γ and decaying learning rates $\gamma_n = Cn^{-\beta}$, for some constants $C > 0$ and $\beta \in (1/2, 1)$. Furthermore, classical decaying learning rates γ_n usually choose a relatively large constant C such as 1. However, as the iteration progresses, the SGD

estimates approach the optimum of convex loss functions, while a large constant in γ_n at this stage may lead to overshooting and slow down the convergence. To mitigate this issue, we propose a mixture learning rate $\tilde{\gamma}_n = \gamma_n - \beta$ with a small constant γ that is close to zero. A burn-in period with the constant learning rate γ in the beginning is adopted to enhance the overall convergence performance. We also provide practical guidelines for tuning the learning rate parameters, including the choice of burn-in periods and the adjustment of mixed learning rates before and after burn-in, based on a target MSE threshold δ . Simulation studies involving linear regression, expectile regression and logistic regression all demonstrate the effectiveness of our proposed methodologies.