THE UNIVERSITY OF CHICAGO

Department of Statistics

BAHADUR MEMORIAL LECTURES

SARA VAN DE GEER

Department of Mathematics
ETH Zürich

Total Variation Regularization Part 1

MONDAY, April 27, 2020, at 4:00 PM
Location: TBA

ABSTRACT

Let $Y \in \mathbb{R}^n$ be a vector of independent observations with unknown mean $f^0 := \mathbb{E}Y$. We consider the estimator $\hat{f} = \hat{f}_D$ that solves the "analysis problem"

$$\min_{f \in \mathbb{R}^n} \left\{ \|Y - f\|_2^2/n + 2\lambda \|Df\|_1 \right\},$$

where $D \in \mathbb{R}^{m \times n}$ is a given matrix and $\lambda > 0$ is a tuning parameter. An example for D is the difference operator

$$(Df)_i = f_i - f_{i-1},\ i \in [2 : n]$$

in which case $\|Df\|_1 = \text{TV}(f)$ is the total variation of f. Other examples include higher order discrete derivatives, total variation on graphs and total variation in higher dimensions. Our aim is to show that the estimator $\hat{f}$ is adaptive. For example, when f^0 is a piecewise linear function, we show that the analysis estimator $\hat{f}_D$, with D the second differences operator, adapts to the number of kinks of f^0. As is the case with the Lasso, the theory for the analysis estimator $\hat{f}$ requires a form of "restricted eigenvalue" condition. We will show that this can be established using interpolating vectors. We will illustrate this (with drawings) for the various examples.

For further information and about building access for persons with disabilities, please contact Jonathan Rodriguez at 773.702.8333 or send email (jgrodriguez@galton.uchicago.edu). If you wish to subscribe to our email list, please visit the following website: https://lists.uchicago.edu/web/arc/statseminars.