SARA VAN DE GEER

Department of Mathematics
ETH Zürich

Total Variation Regularization Part 1

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ABSTRACT

Let $Y \in \mathbb{R}^n$ be a vector of independent observations with unknown mean $f^0 := \mathbb{E}Y$. We consider the estimator $f = f_D$ that solves the “analysis problem”

$$
\min_{f \in \mathbb{R}^n} \left\{ \|Y - f\|_2^2/n + 2\lambda \|Df\|_1 \right\},
$$

where $D \in \mathbb{R}^{m \times n}$ is a given matrix and $\lambda > 0$ is a tuning parameter. An example for $D$ is the difference operator

$$(Df)_i = f_i - f_{i-1}, \ i \in [2 : n]$$

in which case $\|Df\|_1 = \text{TV}(f)$ is the total variation of $f$. Other examples include higher order discrete derivatives, total variation on graphs and total variation in higher dimensions. Our aim is to show that the estimator $f$ is adaptive. For example, when $f^0$ is a piecewise linear function, we show that the analysis estimator $f_D$, with $D$ the second differences operator, adapts to the number of kinks of $f^0$. As is the case with the Lasso, the theory for the analysis estimator $f$ requires a form of “restricted eigenvalue” condition. We will show that this can be established using interpolating vectors. We will illustrate this (with drawings) for the various examples.