Singularities of solutions of the Hamilton-Jacobi equation.
A toy model: distance to a closed subset.

by
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This is a joint work with Piermarco Cannarsa and Wei Cheng.
The distance function $d_F$ to a closed subset $F$ of Euclidean space $\mathbb{R}^k$ is given by

$$d_F(x) = \inf_{f \in F} \|x - f\|.$$ 

It is a Lipschitz, hence differentiable almost everywhere. We will discuss some topological properties of the set $\text{Sing}(d_F)$ of points where $d_F$ is not differentiable.

More generally, we will discuss properties of the set of singularities of a viscosity solution of the Hamilton-Jacobi equation

$$\partial_t U + H(x, \partial_x U) = 0,$$

when $H$ is a Tonelli Hamiltonian.
We will give applications in Riemannian geometry.
We will explain during the lecture all notions (beyond common knowledge) necessary to understand it.