Mathematics Department Paris Program, Spring 2017

Math 29523 (Weeks 1–3): Hyperbolic Geometry and Discrete Groups

Instructor: Howard Masur

In the 19th Century, mathematician such as Bolyai and Lobachevsky developed geometries in which Euclid's Fifth Postulate (the Parallel Postulate) does not hold. The most frequently used models are the disc and the upper half-plane due to Poincaré. This course will study these models of hyperbolic geometry. We will introduce some basic complex analysis and ideas of curvature in geometry to study hyperbolic space. In addition, we will study discrete groups of isometries acting on hyperbolic space and how they give rise to surfaces with negative curvature.

Math 29512 (Weeks 4–6): Introduction to p-Groups

Instructor: Jitka Stehnova

This course is an introduction to p-groups, which play an important role in solvable groups and Lie Algebras. Beginning from the Sylow structure of groups, we will study commutators, the Frattini subgroup, automorphisms, and central products. The course will include a project. The level of difficulty of the project chosen will determine whether this course may be substituted for Math 25600 or Math 25900 in the BS program.

Math 29524 (Weeks 7–9): Elliptic Integrals, Generating Functions, and π

Instructor: Frank Calegari

One learns early on that not all integrals can be evaluated in terms of familiar functions (such as log, exp, trig functions, and polynomials). However, there is a zoo of new and interesting functions in mathematics, many of which satisfy wonderful properties and admit special values. For example, Euler's Gamma function

$$\Gamma(s) = \int_0^\infty e^{-x} x^{s-1}\, dx$$

is an infinitely-differentiable function for which $\Gamma(n+1) = n!$ when n is a natural number. It is also possible to compute special values such as $\Gamma(\frac{1}{2}) = \sqrt{\pi}$.

When one studies infinite series, one learns examples which can be evaluated exactly (such as geometric series), but here too one quickly finds interesting series which have no closed form, such as:

$$\theta(q) = \sum_{-\infty}^{\infty} q^{n^2} = 1 + 2q + 2q^4 + 2q^9 + \cdots$$

In this course, we shall discuss a special class of these functions, and discuss the relationship between this special values and so-called elliptic integrals. These functions have deep links to arithmetic, algebra, and analysis. Sometimes there are special values which can be computed explicitly, for example, one has the following beautiful formula:

$$\theta(e^{-\pi}) = 1 + 2e^{-\pi} + 2e^{-4\pi} + 2e^{-9\pi} + \cdots = \frac{\pi^{1/4}}{\Gamma(3/4)}$$

As an application of these ideas, we will describe an algorithm that will compute π to 45,000,000 decimal places in just 24 steps.