JEFF at 66 ± 1

Pierre Ramond
Institute for Fundamental Theory
University of Florida
At Yale Feza Gürsey convinced me that Exceptional Groups were so beautiful that Nature simply had to use them.

When Jeff searched for an adviser, I suggested he look at these groups because they had no quartic invariants, and their renormalizable potential would have a greater global symmetry, to be broken by gauge interactions. In no time, Jeff came back with an elegant analysis the pseudo Nambu-Goldstone bosons in Exceptional gauge theories.

I stopped at $E_6$ - later Jeff went on to $E_8 \times E_8$: a clear example of Genetic Improvement:

Harvey-R-Reiss 1980-81 “Damn the Torpedoes” papers

A “Compleat” $SO(10)$ Grand Unified Model with CP violation, neutrino masses, from electroweak to Planck scale, without worrying too much about the inelegant underside of the Higgs sector.

Symmetric Texture:

$$ Y^{(-1/3)} = \begin{pmatrix} 0 & R & 0 \\ R & S & 0 \\ 0 & 0 & T \end{pmatrix}, \quad Y^{(-1)} = \begin{pmatrix} 0 & R & 0 \\ R & -3S & 0 \\ 0 & 0 & T \end{pmatrix}, \quad Y^{2/3} = \begin{pmatrix} 0 & P & 0 \\ P & 0 & Q \\ 0 & Q & V \end{pmatrix} $$

Iconoclastic Conclusions ($m_t = 30 \text{ GeV}$):

- Validates Georgi-Jarlskog’s $SU(5)$ Construction
- “Long-Lived” b quark $\tau_B = 4.45 \times 10^{-15} (m_t/m_c) \text{ sec}$
- Neutrino Masses via Seesaw
- Different Quark and Neutrino Mixing Patterns
- One Large Neutrino Mixing Angle
Lepton Mixing Matrix

\[ \Delta I_w = \frac{1}{2} \text{ Charged Leptons Yukawa Matrix} \rightarrow U^{(-1)} \]

\[ \Delta I_w = 0 \text{ Seesaw Majorana Matrix} \rightarrow U_{\text{Seesaw}} \]

\[ U_{\text{PMNS}} = U^{(-1)\dagger} U_{\text{Seesaw}} \]

Two Large Neutrino Mixing Angles

Geometrical Approximation (Harrison, Perkins, Scott)

\[ U_{\text{PMNS}} = \frac{1}{\sqrt{3}} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \equiv U_{\text{TBM}} \]

Hint of Simplicity and Beauty - Flavor Crystal?

Third (reactor) angle measured about 2/3 the Cabibbo angle

Save Beauty and TBM : \[ U_{\text{PMNS}} = U^{(-1)\dagger} U_{\text{TBM}} \]

Only if \( U^{(-1)} \) provides the necessary “Cabibbo Haze”
Generic Georgi-Jarlskog Symmetric Texture

\[
Y^{(-1/3)} = \begin{pmatrix}
0 & a\lambda^3 & b\lambda^3 \\
(1/3) & c\lambda^2 & g\lambda^2 \\
b\lambda^3 & g\lambda^2 & 1
\end{pmatrix}, \quad Y^{(-1)} = \begin{pmatrix}
0 & a\lambda^3 & b\lambda^3 \\
(1/3) & -3c\lambda^2 & g\lambda^2 \\
b\lambda^3 & g\lambda^2 & 1
\end{pmatrix}
\]

**SU(5) Symmetric Texture:**

\[U^{(-1)} = U_{\text{CKM}}(c \to -3c),\]

GUT Scale: \[m_\tau = m_b, \quad m_\mu = 3m_s, \quad m_e = \frac{1}{3}m_d, \quad \rightarrow \quad \det Y^{(-1/3)} = \det Y^{(-1)}\]

Wolfenstein Parameters:

\[a = c = \frac{1}{3}, \quad b = 0.306, \quad g = 0.811, \quad \lambda = \sqrt{m_d/m_s} = \tan \theta_{\text{Cabibbo}}\]

Assume

\[U_{\text{PMNS}} = U_{\text{CKM}}^{(-1)}U_{\text{TBM}}\]

Extract Neutrino’s Third Mixing Angle

\[|\sin \theta_{\text{reactor}}| = \frac{1}{\sqrt{2}} |U_{21}^{(-1)} + U_{31}^{(-1)}| \approx 0.051 \ll \text{Experiment} (0.145)\]

Symmetric Texture + SU(5) + TBM Fails
Down the Rabbit Hole (without a rabbit)

Damn The Torpedos (ignore fine-tuning, for now) ... 40 Years Later
(M.H. Rahat, P. R., B. Xu, Phys. Rev. D 98 (2018)[1805,10684])

Invent an Asymmetric Texture with same Georgi-Jarlskog SU(5)
GUT-Scale relations

\[
Y^{(2/3)} = \begin{pmatrix}
\lambda^8 & 0 & 0 \\
0 & \lambda^4 & 0 \\
0 & 0 & 1
\end{pmatrix}, \quad Y^{(-1/3)} = \begin{pmatrix}
bd\lambda^4 & a\lambda^3 & b\lambda^3 \\
a\lambda^3 & c\lambda^2 & g\lambda^2 \\
d\lambda & g\lambda^2 & 1
\end{pmatrix}, \quad Y^{(-1)} = \begin{pmatrix}
bd\lambda^4 & a\lambda^3 & d\lambda \\
a\lambda^3 & -3c\lambda^2 & g\lambda^2 \\
b\lambda^3 & g\lambda^2 & 1
\end{pmatrix}
\]

\[
d = 2a/g, \quad \text{zero subdeterminant about central element : } \quad \rightarrow \quad \det Y^{(-1/3)} = \det Y^{(-1)}
\]

Asymmetric Texture with TBM Mixing Overshoots $\theta_{\text{reactor}}$

\[
|\sin \theta_{\text{reactor}}| = \frac{\lambda}{3\sqrt{2}} \left(1 + \frac{2}{g}\right) \approx .184 \gg \text{PDG} \ (0.145)
\]
Adding a $\mathcal{CP}$ phase to the TBM matrix lowers $\theta_{\text{reactor}}$

$$U_{\text{TBM}} \rightarrow U_{\text{TBM}}(\delta) = \text{Diag}(1, 1, e^{i\delta}) U_{\text{TBM}}$$

Set $\theta_{\text{reactor}}$ at PDG value and determine the phase:

$$\sin |\theta_{\text{reactor}}| = \frac{\lambda}{3\sqrt{2}} \left| 1 + \frac{2e^{i\delta}}{g} \right| \equiv 0.145 \rightarrow \cos \delta \approx 0.2$$

Asymmetric Texture+ $SU(5)$ Predictions

$$\delta_{CP} = \pm 1.32\pi, \quad \text{compare with global fits:} \quad \delta_{CP}^{PDG} = 1.36^{+0.20}_{-0.16} \pi$$

$$\theta_{\text{Atm}} = 44.9^\circ \ (0.66^\circ \text{ below PDG}), \quad \theta_{\text{Solar}} = 34.16^\circ \ (0.51^\circ \text{ above PDG})$$
$SU(5) \times \mathcal{F}$ Asymmetric Model with Family Symmetry


- All Yukawa Couplings of Dimension 5 and Higher

- $SU(5)$ : Chiral Matter $\bar{5} + 10$ in triplicate

- Vector-Like Messengers

- Familons: Scalar Fields with only Family Symmetry

- Family Symmetry CG Coefficients single out asymmetric terms

- Naturally Vanishing of the Subdeterminant (dim 5 and dim 6)
- Family Symmetry: $\mathcal{F} = T_{13} = (Z_{13} \rtimes Z_3)$

- $Z_{13} \rtimes Z_3 = T_{13} \subset PSL(2, 13) \subset G_2$

- $Z_{13} \rtimes Z_3$ Irreps: $\mathbf{3}_1, \mathbf{3}_2, \mathbf{1}', \overline{\mathbf{3}}_1, \overline{\mathbf{3}}_2, \overline{\mathbf{1}}', \mathbf{1}$

- Asymmetric Chiral Matter: $(\overline{\mathbf{5}}, \mathbf{3}_1) + (\mathbf{10}, \mathbf{3}_2)$

- Higgs fields: $H \sim (\overline{\mathbf{5}}, \mathbf{1}), H' \sim (\overline{\mathbf{45}}, \mathbf{1})$

- Vector-like Messengers:

  $\Delta \sim (\overline{\mathbf{5}}, \mathbf{3}_2)$; explains all charged lepton (down–quarks) Yukawa entries with H

  $\Sigma \sim (\mathbf{10}, \mathbf{3}_1)$ generates the G–J term with H' coupling

  $\Gamma \sim (\overline{\mathbf{10}}, \mathbf{3}_2), \Theta \sim (\overline{\mathbf{10}}, \overline{\mathbf{3}}_1)$ generate the up–quarks Yukawas

- Four familons $\sim (\mathbf{1}, \mathbf{3}_2), \; \text{two familons} \sim (\mathbf{1}, \mathbf{3}_2)$.

- All familon vacuum values along simple directions:

  $(1, 0, 0), (0, 1, 0), (0, 01), (0, 1, 1), (1, 0, 1)$
Sterile Neutrinos: $SU_5$ to $SO_{10}$

$SO_{10} \supset SU_5 \times U(1) : \quad 16 = \bar{5} + 10 + 1; \quad 10_v = \bar{5} + 5$

Vector-Like Mass: $\bar{5}$ from 16 couples with 5 frpm $10_v$

$SU_5 \times T_{13} : (10, \bar{3}_2) + (\bar{5}, 3_1) \rightarrow SO_{10} \times T_{13} : (16, \bar{3}_2) + (10_v, 3_1)$

Right-handed Neutrinos Majorana Mass: $\bar{N} \bar{N} \phi_M, \quad \phi_M \sim 3_2$

With Familon Vacuum: \quad $< \phi_M > = M(1, -1, 1)$

$\mathcal{M}$ diagonalized by TBM

$$\mathcal{U}_{TBM}^t \frac{1}{\mathcal{M}} \mathcal{U}_{TBM} = \frac{1}{M} \text{Diag}(1, -\frac{1}{2}, 1)$$
Seesaw Mechanism

\[ S = \mathcal{D} \frac{1}{\mathcal{M}} \mathcal{D}^t, \]

\( \mathcal{D} \) Dirac mass from \( F \bar{N} H \varphi_D \quad \varphi_D \sim 3_1 \)

With Familon Vacuum : \( \langle \varphi_D \rangle = a(1,-1,1) \)

\( S \) diagonalized by TBM:

\[ S = \frac{a^2}{M} \mathcal{U}_{TBM} \text{Diag}(1,-\frac{1}{2},1) \mathcal{U}_{TBM}^t \]

\[ m_{\nu_1} = m_{\nu_3} = 2m_{\nu_2} \]

No \( (\nu_1 - \nu_3) \) Oscillations?

Are There More Sterile Neutrinos?
Missing Sterile Neutrino: \(SO(10)\) to \(E_6\)

\[ E_6 \supset SO_{10} : \quad 27 = 16 + 10 + 1 \]

\[ SO_{10} \times T_{13} : (16, 3^2) + (\textbf{10}, 3_1) \quad \rightarrow \quad E_6 \times T_{13} : (16, 3^2) + (\textbf{10}, 3_1) + (1, 1') \]

Mixture of \(E_6\) \(27\) with \(PSL(2, 13)\) complex septet: \(7 = 3_1 + 3_2 + 1'\)

With fourth sterile neutrino (using oscillations):

\[ m_{\nu_1} = 27.6 \text{ meV}, \quad m_{\nu_2} = 28.9 \text{ meV}, \quad m_{\nu_3} = 57.8 \text{ meV} \]

Below the Rabbit Hole

Simpler version: \(E_6 \times T_7 : \quad (16, 3) + (\textbf{10}, \bar{3}) + (1, 1)\)

Mixture of \(27\) of \(E_6\) and the real septet of \(PSL(2, 7)\): \(7 = 3 + \bar{3} + 1\)

\(PSL(2, 13)\) and \(PSL(2, 7)\) are modular groups and finite subgroups of \(G_2\)

Octonions? Eleven Dimensions, Moonshine, ...
Neutrino Chronology

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<th>Year</th>
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<td>1930</td>
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<tr>
<td>Detection</td>
<td>$2 \cdot (13)$</td>
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<tr>
<td>Oscillation</td>
<td>$2^2 \cdot (17)$</td>
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<tr>
<td>$\nu \beta \beta$ Decay</td>
<td>$2^3 \cdot (19)$</td>
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