
JEFF at $66 \pm 1$

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At Yale Feza Gürsey convinced me that Exceptional Groups were so beautiful that Nature simply had to use them.

When Jeff searched for an adviser, I suggested he look at these groups because they had no quartic invariants, and their renormalizable potential would have a greater global symmetry, to be broken by gauge interactions. In no time, Jeff came back with an elegant analysis the pseudo Nambu-Goldstone bosons in Exceptional gauge theories.

I stopped at $E_6$ - later Jeff went on to $E_8 \times E_8$: a clear example of Genetic Improvement:

Harvey-R-Reiss 1980-81 “Damn the Torpedoes” papers

A “Compleat” $SO(10)$ Grand Unified Model with CP violation, neutrino masses, from electroweak to Planck scale, without worrying too much about the inelegant underside of the Higgs sector.

Symmetric Texture:

$$Y^{(-1/3)} = \begin{pmatrix} 0 & R & 0 \\ R & S & 0 \\ 0 & 0 & T \end{pmatrix} \quad Y^{(-1)} = \begin{pmatrix} 0 & R & 0 \\ R & -3S & 0 \\ 0 & 0 & T \end{pmatrix}, \quad Y^{2/3} = \begin{pmatrix} 0 & P & 0 \\ P & 0 & Q \\ 0 & Q & V \end{pmatrix}$$

Iconoclastic Conclusions ($m_t = 30\ GeV$):

- Validates Georgi-Jarlskog's $SU(5)$ Construction

- “Long-Lived” b quark $\tau_B = 4.45 \times 10^{-15}(m_t/m_c)$ sec

- Neutrino Masses via Seesaw

- Different Quark and Neutrino Mixing Patterns

- One Large Neutrino Mixing Angle

Lepton Mixing Matrix

$\Delta I_w = \frac{1}{2}$ Charged Leptons Yukawa Matrix $\quad \longrightarrow \quad \mathcal{U}^{(-1)}$

$\Delta I_w = 0$ Seesaw Majorana Matrix $\quad \longrightarrow \quad \mathcal{U}_{\text{Seesaw}}$

$$\mathcal{U}_{\text{PMNS}} = \mathcal{U}^{(-1)\dagger}\mathcal{U}_{\text{Seesaw}}$$

Two Large Neutrino Mixing Angles

Geometrical Approximation (Harrison, Perkins, Scott)

$$\mathcal{U}_{\text{PMNS}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \equiv \mathcal{U}_{\text{TBM}}$$

Hint of Simplicity and Beauty - Flavor Crystal?

Third (reactor) angle measured about 2/3 the Cabibbo angle

Save Beauty and TBM : $\quad \longrightarrow \quad \mathcal{U}_{\text{PMNS}} = \mathcal{U}^{(-1)\dagger}\mathcal{U}_{\text{TBM}}$

Only if $\mathcal{U}^{(-1)}$ provides the necessary “Cabibbo Haze”

## Generic Georgi-Jarlskog Symmetric Texture

$$Y^{(-1/3)} = \begin{pmatrix} 0 & a\lambda^3 & b\lambda^3 \\ a\lambda^3 & c\lambda^2 & g\lambda^2 \\ b\lambda^3 & g\lambda^2 & 1 \end{pmatrix}, \quad Y^{(-1)} = \begin{pmatrix} 0 & a\lambda^3 & b\lambda^3 \\ a\lambda^3 & -3c\lambda^2 & g\lambda^2 \\ b\lambda^3 & g\lambda^2 & 1 \end{pmatrix}$$

$SU(5)$ Symmetric Texture:

$$\mathcal{U}^{(-1)} = \mathcal{U}_{\text{CKM}}(c \;\rightarrow\; -3c),$$

$$\text{GUT}-\text{Scale}: \quad m_\tau = m_b,\ m_\mu = 3m_s,\ m_e = \frac{1}{3}m_d, \quad \longrightarrow \quad \det Y^{(-1/3)} = \det Y^{(-1)}$$

Wolfenstein Parameters:

$$a = c = \frac{1}{3}, \quad b = 0.306, \quad g = 0.811, \quad \lambda = \sqrt{\frac{m_d}{m_s}} = \tan\theta_{\text{Cabibbo}}$$

Assume

$$\mathcal{U}_{\text{PMNS}} = \mathcal{U}^{(-1)\dagger}\mathcal{U}_{\text{TBM}}.$$

Extract Neutrino's Third Mixing Angle

$$|\sin\theta_{\text{reactor}}| = \frac{1}{\sqrt{2}}\left|\mathcal{U}^{(-1)}_{21} + \mathcal{U}^{(-1)}_{31}\right| \approx .051 \ll \text{ Experiment } (0.145)$$

## Symmetric Texture + $SU(5)$ +TBM Fails

## Down the Rabbit Hole (without a rabbit)

## Damn The Torpedos (ignore fine-tuning, for now) ... 40 Years Later

(M.H. Rahat, P. R., B. Xu, Phys. Rev. D **98** (2018)[1805,10684])

Invent an Asymmetric Texture with same Georgi-Jarlskog $SU(5)$ GUT-Scale relations

$$Y^{(2/3)} = \begin{pmatrix} \lambda^8 & 0 & 0 \\ 0 & \lambda^4 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \; Y^{(-1/3)} = \begin{pmatrix} bd\lambda^4 & a\lambda^3 & b\lambda^3 \\ a\lambda^3 & c\lambda^2 & g\lambda^2 \\ d\lambda & g\lambda^2 & 1 \end{pmatrix}, \; Y^{(-1)} = \begin{pmatrix} bd\lambda^4 & a\lambda^3 & d\lambda \\ a\lambda^3 & -3c\lambda^2 & g\lambda^2 \\ b\lambda^3 & g\lambda^2 & 1 \end{pmatrix}$$

$$d = 2a/g, \quad \text{zero subdeterminant about central element} : \;\longrightarrow\; \det Y^{(-1/3)} = \det Y^{(-1)}$$

Asymmetric Texture with TBM Mixing Overshoots $\theta_{\text{reactor}}$

$$|\sin\theta_{\text{reactor}}| = \frac{\lambda}{3\sqrt{2}}(1 + \frac{2}{g}) \approx .184 \gg \text{ PDG } (0.145)$$

Adding a $\not{C}$P phase to the TBM matrix *lowers* $\theta_{\text{reactor}}$

$$\mathcal{U}_{\text{TBM}} \longrightarrow U_{\text{TBM}}(\delta) = \text{Diag}(1, 1, e^{i\delta})\,\mathcal{U}_{\text{TBM}}$$

Set $\theta_{\text{reactor}}$ at PDG value and determine the phase:

$$\sin|\theta_{\text{reactor}}| = \frac{\lambda}{3\sqrt{2}}\left|1 + \frac{2e^{i\delta}}{g}\right| \equiv 0.145 \;\; \rightarrow \cos\delta \approx 0.2$$

## Asymmetric Texture+ $SU(5)$ Predictions

$$\delta_{CP} = \pm 1.32\pi, \quad \text{compare with global fits :} \quad \delta_{CP}^{PDG} = 1.36^{+0.20}_{-0.16}\pi$$

$$\theta_{\text{Atm}} = 44.9^\circ \; (0.66^\circ \; \textit{below PDG}), \quad \theta_{\text{Solar}} = 34.16^\circ \; (0.51^\circ \; \textit{above PDG})$$

# $SU(5) \times \mathcal{F}$ Asymmetric Model with Family Symmetry

(M.J. Pérez, M.H. Rahat, P. R., A.J. Stuart, B. Xu, Phys. Rev. D **100**(2019) 075008; **101**(2020)

- All Yukawa Couplings of Dimension 5 and Higher

- $SU(5)$ : Chiral Matter $\bar{\mathbf{5}} + \mathbf{10}$ in triplicate

- Vector-Like Messengers

- Familons: Scalar Fields with only Family Symmetry

- Family Symmetry CG Coefficients single out asymmetric terms

- Naturally Vanishing of the Subdeterminant (dim 5 and dim 6)

- Family Symmetry : $\mathcal{F} = T_{13} = (Z_{13} \rtimes Z_3)$

- $Z_{13} \rtimes Z_3 = T_{13} \subset PSL(2, 13) \subset G_2$

- $Z_{13} \rtimes Z_3$ Irreps : $\mathbf{3_1}, \mathbf{3_2}, \mathbf{1'}, \mathbf{\bar{3}_1}, \mathbf{\bar{3}_2}, \mathbf{\bar{1}'}, \mathbf{1}$

- Asymmetric Chiral Matter : $(\mathbf{\bar{5}}, \mathbf{3_1}) + (\mathbf{10}, \mathbf{3_2})$

- Higgs fields :$H \sim (\mathbf{\bar{5}}, \mathbf{1})$, $H' \sim (\overline{\mathbf{45}}, \mathbf{1})$

- Vector − like Messengers :

$\Delta \sim (\mathbf{5}, \mathbf{3_2})$; explains all charged lepton (down − quarks) Yukawa entries with H

$\Sigma \sim (\mathbf{10}, \mathbf{3_1})$ generates the G − J term with H′ coupling

$\Gamma \sim (\overline{\mathbf{10}}, \mathbf{3_2})$, $\Theta \sim (\overline{\mathbf{10}}, \mathbf{\bar{3}_1})$ generate the up − quarks Yukawas

- Four familons $\sim (\mathbf{1}, \mathbf{3_2})$, two familons $\sim (\mathbf{1}, \mathbf{3_2})$.

- All familon vacuum values along simple directions :

$(1, 0, 0), (0, 1, 0), (0, 01), (0, 1, 1, ), (1, 0, 1)$

## Sterile Neutrinos: $SU_5$ to $SO_{10}$

$$SO_{10} \supset SU_5 \times U(1): \qquad \mathbf{16} = \bar{\mathbf{5}} + \mathbf{10} + 1; \quad \mathbf{10}_v = \bar{\mathbf{5}} + \mathbf{5}$$

Vector-Like Mass: $\bar{\mathbf{5}}$ from **16** couples with **5** frpm $\mathbf{10}_v$

$$SU_5 \times T_{13}: \ (\mathbf{10}, \mathbf{3_2}) + (\bar{\mathbf{5}}, \mathbf{3_1}) \ \longrightarrow \ SO_{10} \times T_{13}: \ (\mathbf{16}, \mathbf{3_2}) + (\mathbf{10}_v, \mathbf{3_1})$$

$$\mathrm{Right-handed\ Neutrinos\ Majorana\ Mass:} \, \bar{\mathrm{N}}\,\bar{\mathrm{N}}\,\varphi_{\mathcal{M}}, \quad \varphi_{\mathcal{M}} \sim \mathbf{3_2}$$

$$\mathrm{With\ Familon\ Vacuum:} \quad <\varphi_{\mathcal{M}}> = M(1,-1,1)$$

## $\mathcal{M}$ diagonalized by TBM

$$\mathcal{U}^t_{TBM} \frac{1}{\mathcal{M}} \mathcal{U}_{TBM} = \frac{1}{M} \mathrm{Diag}(1, -\frac{1}{2}, 1)$$

# Seesaw Mechanism

$$\mathcal{S} = \mathcal{D}\frac{1}{\mathcal{M}}\mathcal{D}^t,$$

$\mathcal{D}$ Dirac mass from $\mathrm{F}\bar{\mathrm{N}}\,\mathrm{H}\,\varphi_{\mathcal{D}}$ $\quad \varphi_{\mathcal{D}} \sim \bar{\mathbf{3}}_{\mathbf{1}}$

With Familon Vacuum : $\; < \varphi_{\mathcal{D}} > = a(1, -1, 1)$

$\mathcal{S}$ diagonalized by TBM:

$$\mathcal{S} = \frac{a^2}{M}\mathcal{U}_{\mathrm{TBM}}\mathrm{Diag}(1, -\frac{1}{2}, 1)\mathcal{U}^t_{\mathrm{TBM}}$$

$$m_{\nu_1} = m_{\nu_3} = 2m_{\nu_2}$$

No $(\nu_1 - \nu_3)$ Oscillations?

Are There More Sterile Neutrinos?

Missing Sterile Neutrino : SO(10) to $\mathrm{E}_6$

$$E_6 \supset SO_{10} : \qquad \mathbf{27} = \mathbf{16} + \mathbf{10} + \mathbf{1}$$

$$SO_{10}{\times}T_{13} : \ (\mathbf{16}, \mathbf{3_2}){+}(\mathbf{10}, \mathbf{3_1}) \quad \longrightarrow \quad E_6{\times}T_{13} : \ (\mathbf{16}, \mathbf{3_2}){+}(\mathbf{10}, \mathbf{3_1}){+}(\mathbf{1}, \mathbf{1'})$$

Mixture of $E_6$ **27** with $PSL(2,13)$ complex septet : $\mathbf{7} = \mathbf{3_1}{+}\mathbf{3_2}{+}\mathbf{1'}$

With fourth sterile neutrino (using oscillations):

$$m_{\nu_1} = 27.6 \text{ meV}, \quad m_{\nu_2} = 28.9 \text{ meV}, \quad m_{\nu_3} = 57.8 \text{ meV}$$

Below the Rabbit Hole

Simpler version : $E_6 \times T_7$ : $\qquad (\mathbf{16}, \mathbf{3}) + (\mathbf{10}, \bar{\mathbf{3}}) + (\mathbf{1}, \mathbf{1})$

Mixture of **27** of $E_6$ and the real septet of $PSL(2,7)$ : $\mathbf{7} = \mathbf{3}{+}\bar{\mathbf{3}}{+}\mathbf{1}$

$PSL(2,13)$ and $PSL(2,7)$ are modular groups and finite subgroups of $G_2$

Octonions? Eleven Dimensions, Moonshine, ...

# Neutrino Chronology

| | | |
|---|---|---|
| Revelation | | 1930 |
| Detection | $2 \cdot (13)$ | 1956 |
| Oscillation | $2^2 \cdot (17)$ | 1998 |
| $\not\nu\beta\beta$Decay | $2^3 \cdot (19)$ | 2052 |