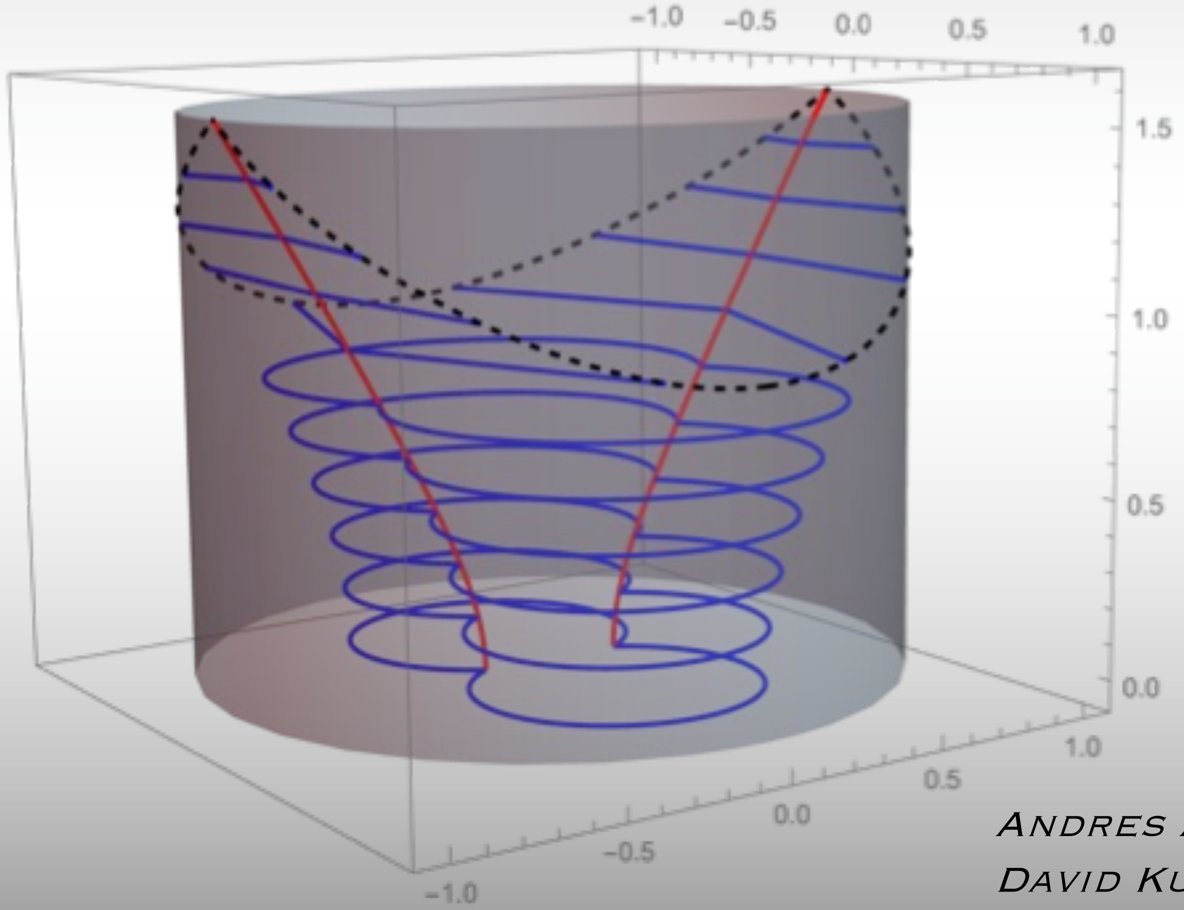
Accelerating Black Holes



Ruth Gregory

Jeff-Fest

13-5-22

Andres Anabalon, Mike Appels, Finn Gray, David Kubiznak, Rob Mann, Ali Ovgun, Andy Scoins, Gabriel Arenas-Henriquez

Happy Birthday Jeff!

Cinderella Strings

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Spacetime decay of cones at strong coupling

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Unwinding Strings and T-duality of Kaluza-Klein and H-Monopoles

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BLACK HOLES WITH A MASSIVE DILATON

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QUANTUM FERMION HAIR

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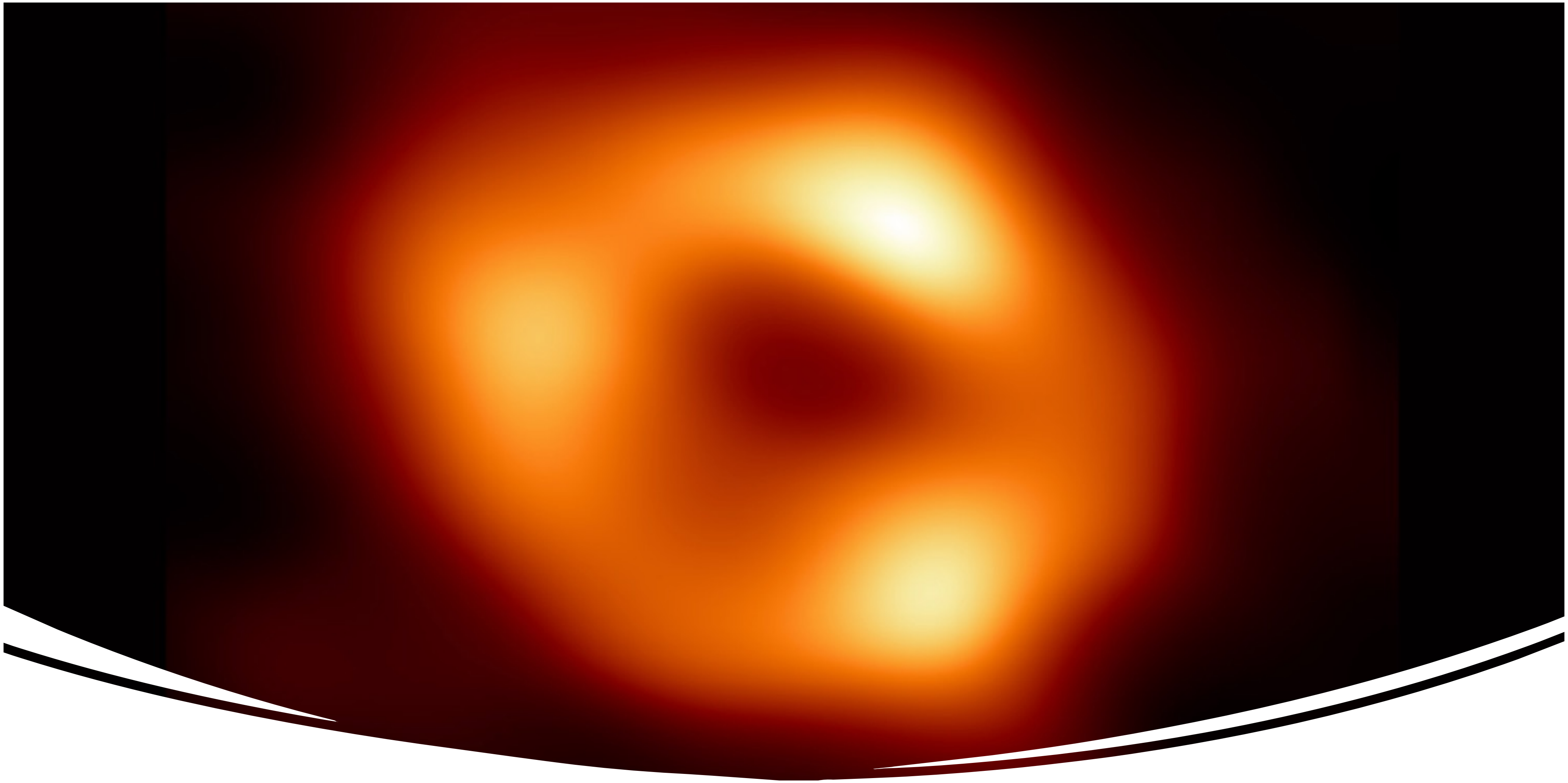


aCceleration

- C in 4
- Some chemistry (!)
- A theorem! (Though not C)
- C in 3

Acceleration in 4D

An accelerating black hole in 4D is described by the C-metric

$$ds^2 = \Omega^{-2}\left[f(r)dt^2 - \frac{dr^2}{f(r)} - r^2\left(\frac{d\theta^2}{g(\theta)} + g(\theta)\sin^2\theta\frac{d\phi^2}{K^2}\right)\right]$$

Where

$$f = \left(1 - \frac{2m}{r}\right)\left(1 - A^2r^2\right) + \frac{r^2}{\ell^2}$$

$$g = 1 + 2mA\cos\theta$$

$$\Omega = 1 + Ar\cos\theta$$

f determines horizon structure – black hole / acceleration / cosmological constant

Hong & Teo, CQG20 3629 (2003)

Conical Deficits

Start by thinking about deficits. Known exact solutions have a symmetry axis, and we can "cut out" a portion of the angular coordinate – this is a legitimate GR solution and can be sourced by a cosmic string.

$$ds^2_{\theta,\phi} \propto d\theta^2 + \frac{\theta^2}{K^2} d\phi^2$$

K measures a deficit or "cosmic string" on axis, keeping the angular coordinate at fixed periodicity

$$\delta = 2\pi \left(1 - \frac{1}{K}\right) = 8\pi\mu$$

c.f. Aryal, Ford, Vilenkin: PRD 34, 2263 (1986), Achucarro, Gregory, Kuijken: PRD 52 5729 (1995)

Thermodynamics, with strings attached!

Want to do thermodynamics with strings, so start with a semi-familiar case: Schwarzschild-AdS with a deficit: $f = 1-2m/r+r^2/\ell^2$

Black hole horizon defined by f=0, look at small changes in f. Horizon still defined by f(r) = 0.

$$f(r_+ + \delta r_+) = f'(r_+)\delta r_+ + \frac{\partial f}{\partial m}\delta m + \frac{\partial f}{\partial \ell}\delta \ell = 0$$

Changes r+, hence S

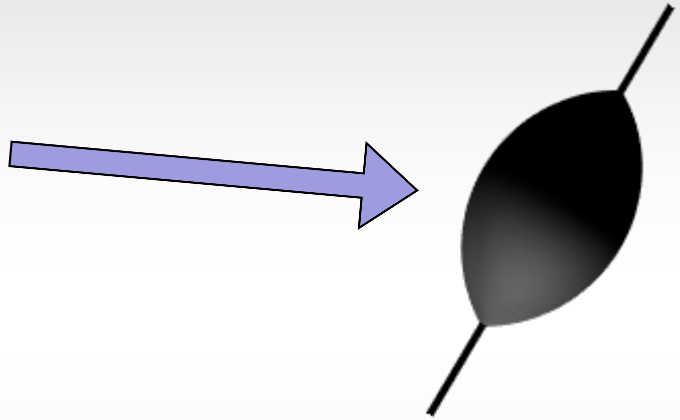
Changes m, hence M

Changes ℓ, hence Λ

Black Hole Thermodynamics

Temperature has usual definition, but entropy depends on K:

$$T = \frac{f'(r_+)}{4\pi} \qquad S = \frac{\pi r_+^2}{K}$$



-and we want to do thermodynamics including the string, so we have to take into account varying K.

$$\delta S = \frac{\pi r_+ \delta r_+}{K} - \pi r_+^2 \frac{\delta K}{K^2}$$

(S): Herdeiro, Kleihaus,Kunz,Radu: 0912:3386 [gr-qc]

Changing Tension

Tension is related to K:

$$\mu = \frac{1}{4}\left(1 - \frac{1}{K}\right)$$

So easily get

$$\delta\mu = \frac{\delta K}{4K^2}$$

Finally

$$P = -\Lambda = \frac{3}{8\pi\ell^2} \qquad V = \frac{4\pi r_+^3}{3K}$$

First Law with Tension

Putting together:

$$0 = \frac{2K}{r_+}\left(T\delta S + 2(m - r_+)\delta\mu + V\delta P - \delta(\frac{m}{K})\right)$$

So identify

$$M = \frac{m}{K}$$

Then also get Smarr relation:

$$M = 2TS - 2PV$$

Thermodynamic Length

The term multiplying the variation in tension is a "thermodynamic length"

$$\lambda = r_+ - m$$

Reinforces interpretation of M as **enthalpy**, if black hole grows, it swallows some string, but has also displaced the same amount of energy from environment.

Kastor & Traschen: 1207:5415 [hep-th]

Acceleration

To get acceleration (A) add different tensions for N and S strings:

$$\delta_{\pm} = 2\pi\left(1 - \frac{g(0)}{K}\right) = 2\pi\left(1 - \frac{1 \pm 2mA}{K}\right) = \text{“}8\pi\mu_{\pm}\text{”}$$

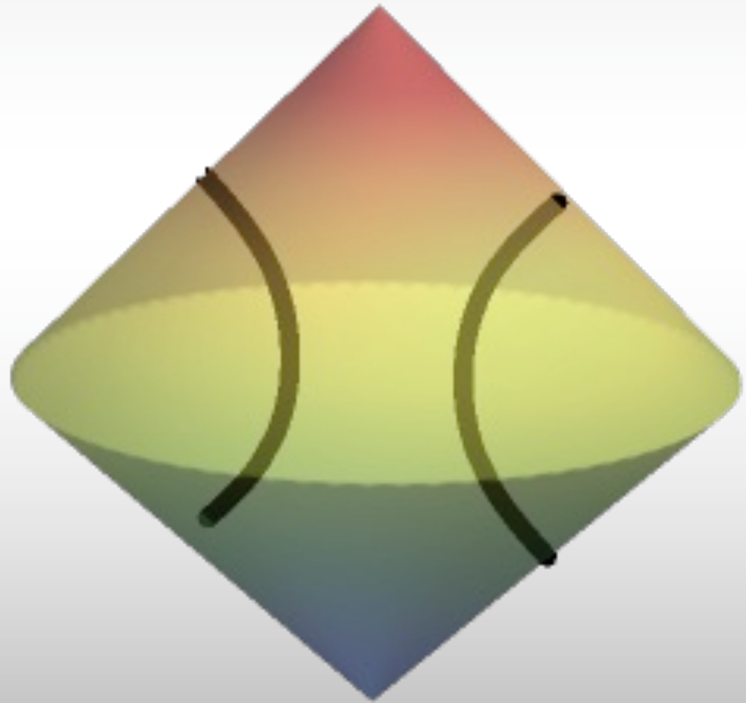
Conventionally, we make N axis regular, with deficit on S axis

$$\delta_N = 0 \Rightarrow K = 1 + 2mA$$

$$\delta_S = \frac{8\pi mA}{K} \Rightarrow \mu_S = \frac{mA}{K}$$

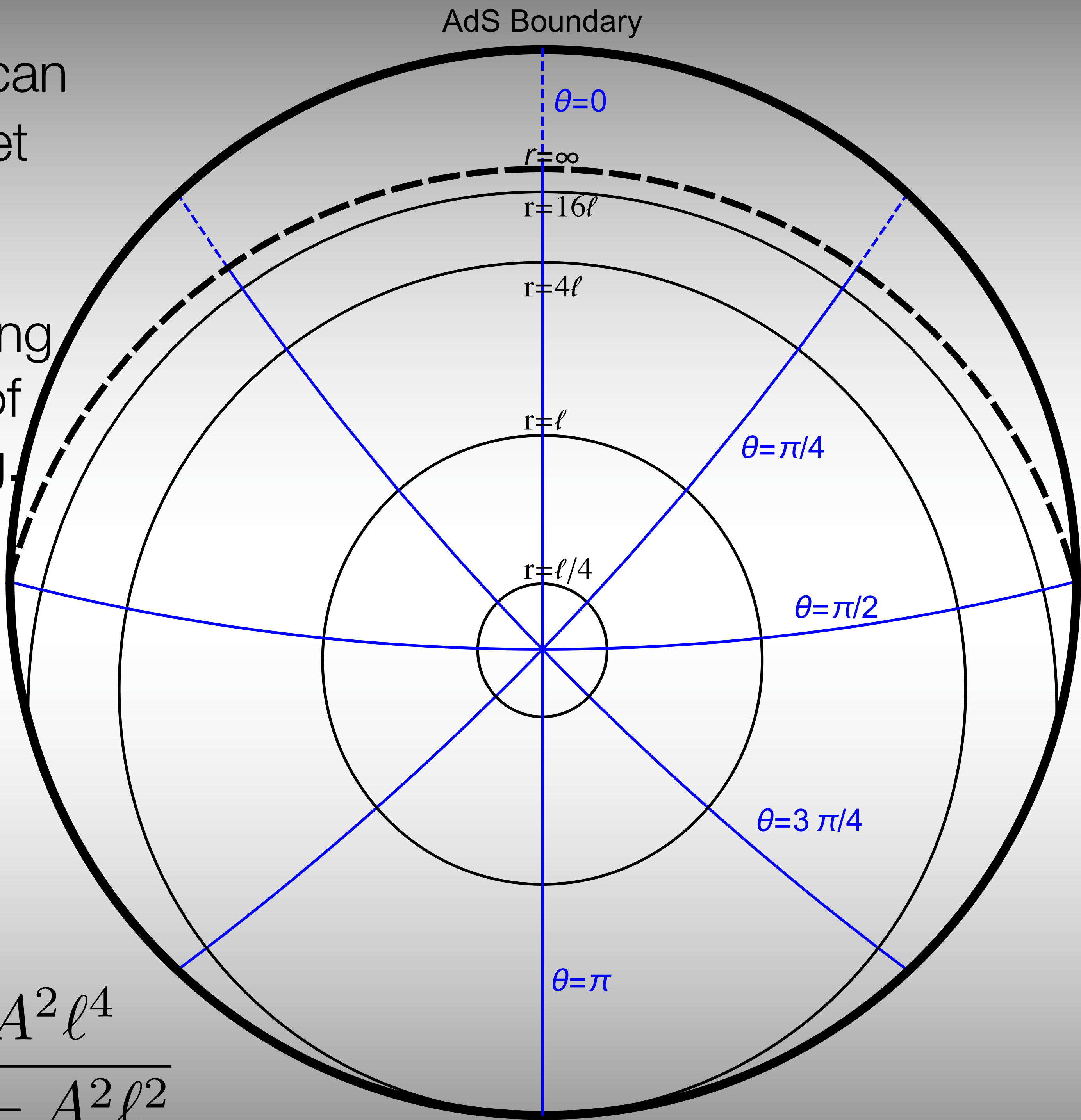
General C-metrics

In general, the C-metric has acceleration horizon, so thermodynamics would refer locally to black hole horizon – here we can see the nontrivial nature of the spacetime more readily.



We look at slowly accelerating black holes to have only 1 horizon.

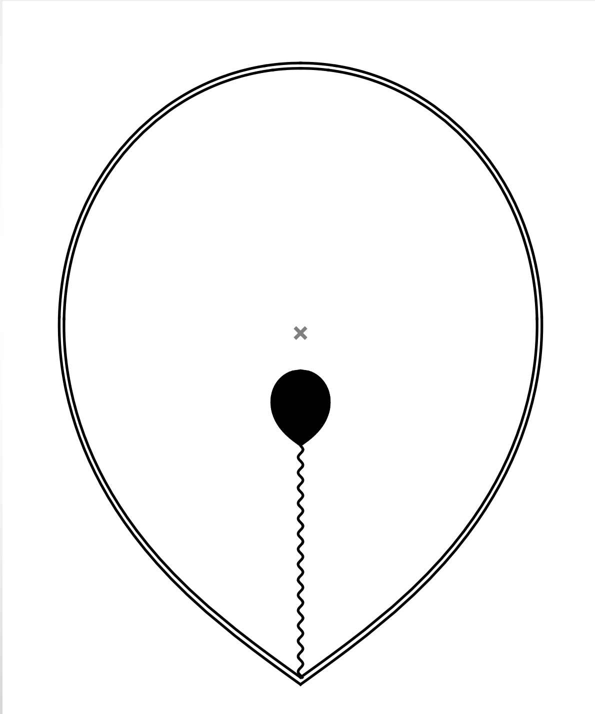
If m=0 and Aℓ <1, can change cords to get AdS from an off-centre perspective. An observer hovering away from centre of AdS is accelerating. This is SLOWLY ACCELERATING RINDLER



$$r = 0 \leftrightarrow R = \frac{A^2\ell^4}{1 - A^2\ell^2}$$

The Slowly Accelerating Black Hole

The slowly accelerating black hole in AdS is displaced from centre. It has a conical deficit running from the horizon to the boundary. The string tension provides the force that hold the black hole off-centre.



Return to the Metric

Based on experience with the Kerr-AdS metric (and motivated by the coordinate transformation for slowly accelerating Rindler) we introduce a possible rescaling of the time coordinate

$$ds^2 = \frac{1}{H^2}\left\{\frac{f(r)}{\Sigma}\left[\frac{dt}{\alpha} - a\sin^2\theta\frac{d\varphi}{K}\right]^2 - \frac{\Sigma}{f(r)}dr^2 - \frac{\Sigma r^2}{h(\theta)}d\theta^2 - \frac{h(\theta)\sin^2\theta}{\Sigma r^2}\left[\frac{a\,dt}{\alpha} - (r^2+a^2)\frac{d\varphi}{K}\right]^2\right\}$$

This will rescale temperature, and also changes computations of the mass.

$$f(r) = (1 - A^2 r^2)\left[1 - \frac{2m}{r} + \frac{a^2+e^2}{r^2}\right] + \frac{r^2+a^2}{\ell^2}\,,$$

$$h(\theta) = 1 + 2mA\cos\theta + \left[A^2(a^2+e^2) - \frac{a^2}{\ell^2}\right]\cos^2\theta\,,$$

$$\Sigma = 1 + \frac{a^2}{r^2}\cos^2\theta\,, \qquad H = 1 + Ar\cos\theta\,.$$

CHECK M, T AND S

Using the usual Euclidean method, find temperature:

$$T = \frac{f'_+}{4\pi\alpha} = \frac{1}{2\pi r_+^2 \alpha}\left[m(1 - A^2 r_+^2) + \frac{r_+^3}{\ell^2(1 - A^2 r_+^2)}\right]$$

Which depends on alpha, and entropy:

$$S = \frac{\pi r_+^2}{K(1 - A^2 r_+^2)}$$

What is M?

Expand the metric near the boundary (Fefferman-Graham):

$$\frac{1}{r} = -A\xi - \sum F_n(\xi)\, z^n$$
$$\cos\theta = \xi + \sum G_n(\xi)\, z^n$$

F_n and G_n determined by the requirement that

$$ds^2 = -\ell^2 dz^2 + \frac{1}{z^2}\left[\gamma_{\mu\nu} + z^2\Psi_{\mu\nu} + z^3 M_{\mu\nu}\right] dx^\mu dx^\nu + \mathcal{O}(z^2)$$

Fefferman-Graham

For the boundary metric, get:

$$\frac{\left(1-A^2\ell^2 g(\xi)\right)^3}{\alpha^2\ell^2 F_1^2(\xi)}d\tau^2 - \frac{\left(1-A^2\ell^2 g(\xi)\right)}{F_1^2(\xi)g(\xi)}d\xi^2 - \frac{g(\xi)\left(1-A^2\ell^2 g(\xi)\right)^2}{K^2 F_1^2(\xi)}d\phi^2$$

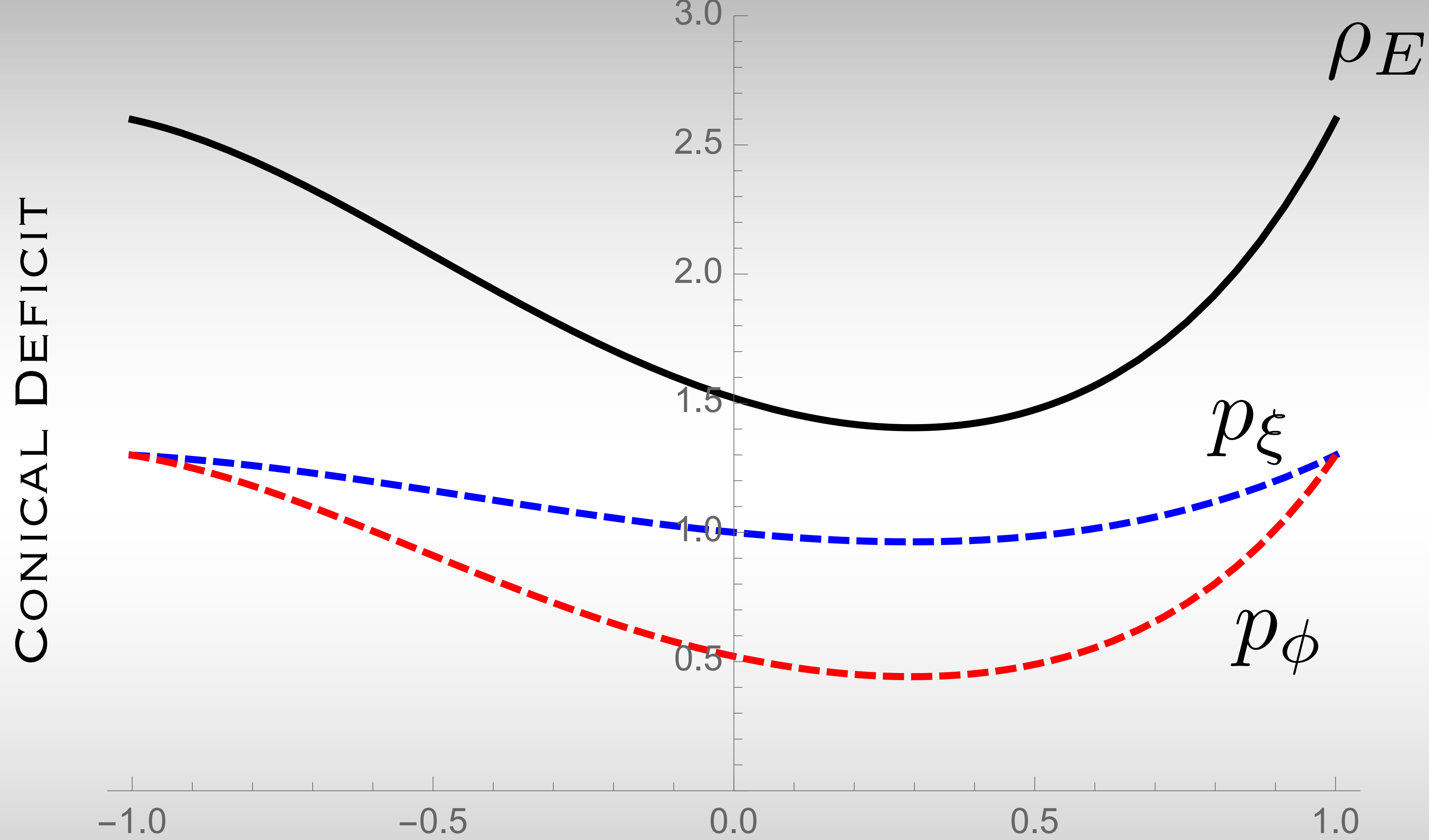
And for the boundary fluid stress tensor:

$$\langle \mathcal{T}_\nu^\mu \rangle = \mathrm{diag}\left\{\rho_E, -\rho_E/2+\Pi, \rho_E/2-\Pi\right\}$$

where

$$\rho_E = \frac{m}{\alpha}(1-A^2\ell^2 g)^{3/2}(2-3A^2\ell^2 g)$$

$$\Pi = \frac{3A^2\ell^2 gm}{2\alpha}(1-A^2\ell^2 g)^{3/2}$$



Conical Deficit
3.0
2.5
2.0
1.5
1.0
0.5
−1.0
−0.5
0.0
0.5
1.0
ρ_E
p_ξ
p_ϕ

Accelerating Thermodynamics

Integrate up the boundary stress-energy to get the mass:

$$M = \int \rho_E \sqrt{\gamma} = \frac{\alpha m}{K}$$

What is alpha? Setting m to zero, and demanding that the boundary is a round 2-sphere gives

$$\alpha = \sqrt{1 - A^2 \ell^2}$$

Get a consistent first law with corrections to V and TD length, and – can generalise to rotation

General Thermo Parameters

$$M = \frac{m(\Xi + a^2/\ell^2)(1 - A^2\ell^2\Xi)}{K\Xi\alpha(1 + a^2A^2)}$$

$$T = \frac{f'_+ r_+^2}{4\pi\alpha(r_+^2 + a^2)}\,, \quad S = \frac{\pi(r_+^2 + a^2)}{K(1 - A^2 r_+^2)}\,,$$

$$Q = \frac{e}{K}\,, \quad \Phi = \Phi_t = \frac{e r_+}{(r_+^2 + a^2)\alpha}\,,$$

$$J = \frac{ma}{K^2}\,, \quad \Omega = \Omega_H - \Omega_\infty\,, \quad \Omega_H = \frac{Ka}{\alpha(r_+^2 + a^2)}$$

$$P = \frac{3}{8\pi\ell^2}\,, \quad V = \frac{4\pi}{3K\alpha}\left[\frac{r_+(r_+^2 + a^2)}{(1 - A^2 r_+^2)} + \frac{m[a^2(1 - A^2\ell^2\Xi) + A^2\ell^4\Xi(\Xi + a^2/\ell^2)]}{(1 + a^2A^2)\Xi}\right]$$

$$\lambda_\pm = \frac{r_+}{\alpha(1 \pm A r_+)} - \frac{m}{\alpha}\frac{[\Xi + a^2/\ell^2 + \frac{a^2}{\ell^2}(1 - A^2\ell^2\Xi)]}{(1 + a^2A^2)\Xi^2} \mp \frac{A\ell^2(\Xi + a^2/\ell^2)}{\alpha(1 + a^2A^2)}$$

$$\Xi = 1 - \frac{a^2}{l^2} + A^2(e^2 + a^2) \qquad \alpha = \frac{\sqrt{(\Xi + a^2/\ell^2)(1 - A^2\ell^2\Xi)}}{1 + a^2A^2}$$

Chemical Expressions

It is helpful to draw analogies with Chemistry, expressing the chemical potentials in terms of thermodynamical charges, rather than geometrical quantities like r_+.
A starting point is the Christodoulou-Ruffini mass formula:

$$M^2(S,Q,J,P) = \frac{S}{4\pi}\left[\left(1+\frac{\pi Q^2}{S}+\frac{8PS}{3}\right)^2+\left(1+\frac{8PS}{3}\right)\frac{4\pi^2 J^2}{S^2}\right]$$

Here shown for Kerr-Newman-AdS.

The Deficits

While we originally derived the thermodynamics for each deficit separately, it is more natural to think in terms of an overall average deficit, and the differential deficit that produces acceleration. We therefore encode:

$$\Delta = 1 - 2(\mu_+ + \mu_-) = \frac{\Xi}{K}$$

$$C = \frac{(\mu_- - \mu_+)}{\Delta} = \frac{mA}{K\Delta} = \frac{mA}{\Xi}$$

Then, by observing how the charges/potentials scaled with K, conjectured how the C-R formula would generalise.

$$\left(\Xi = 1 + e^2A^2 - \frac{a^2}{\ell^2}(1 - A^2\ell^2)\right)$$

$$M^2 = \frac{\Delta S}{4\pi}\left[\left(1+\frac{\pi Q^2}{\Delta S}+\frac{8PS}{3\Delta}\right)^2+\left(1+\frac{8PS}{3\Delta}\right)\left(\frac{4\pi^2 J^2}{(\Delta S)^2}-\frac{3C^2\Delta}{2PS}\right)\right]$$

$$V = \frac{2S^2}{3\pi M}\left[\left(1+\frac{\pi Q^2}{\Delta S}+\frac{8PS}{3\Delta}\right)+\frac{2\pi^2 J^2}{(\Delta S)^2}+\frac{9C^2\Delta^2}{32P^2S^2}\right],$$

$$T = \frac{\Delta}{8\pi M}\left[\left(1+\frac{\pi Q^2}{\Delta S}+\frac{8PS}{3\Delta}\right)\left(1-\frac{\pi Q^2}{\Delta S}+\frac{8PS}{\Delta}\right)-\frac{4\pi^2 J^2}{(\Delta S)^2}-4C^2\right],$$

$$\Omega = \frac{\pi J}{SM\Delta}\left(1+\frac{8PS}{3\Delta}\right),$$

$$\Phi = \frac{Q}{2M}\left(1+\frac{\pi Q^2}{S\Delta}+\frac{8PS}{3\Delta}\right),$$

$$\lambda_\pm = \frac{S}{4\pi M}\left[\left(\frac{8PS}{3\Delta}+\frac{\pi Q^2}{\Delta S}\right)^2+\frac{4\pi^2 J^2}{(\Delta S)^2}\left(1+\frac{16PS}{3\Delta}\right)-(1\mp 2C)^2\pm\frac{3C\Delta}{2PS}\right]$$

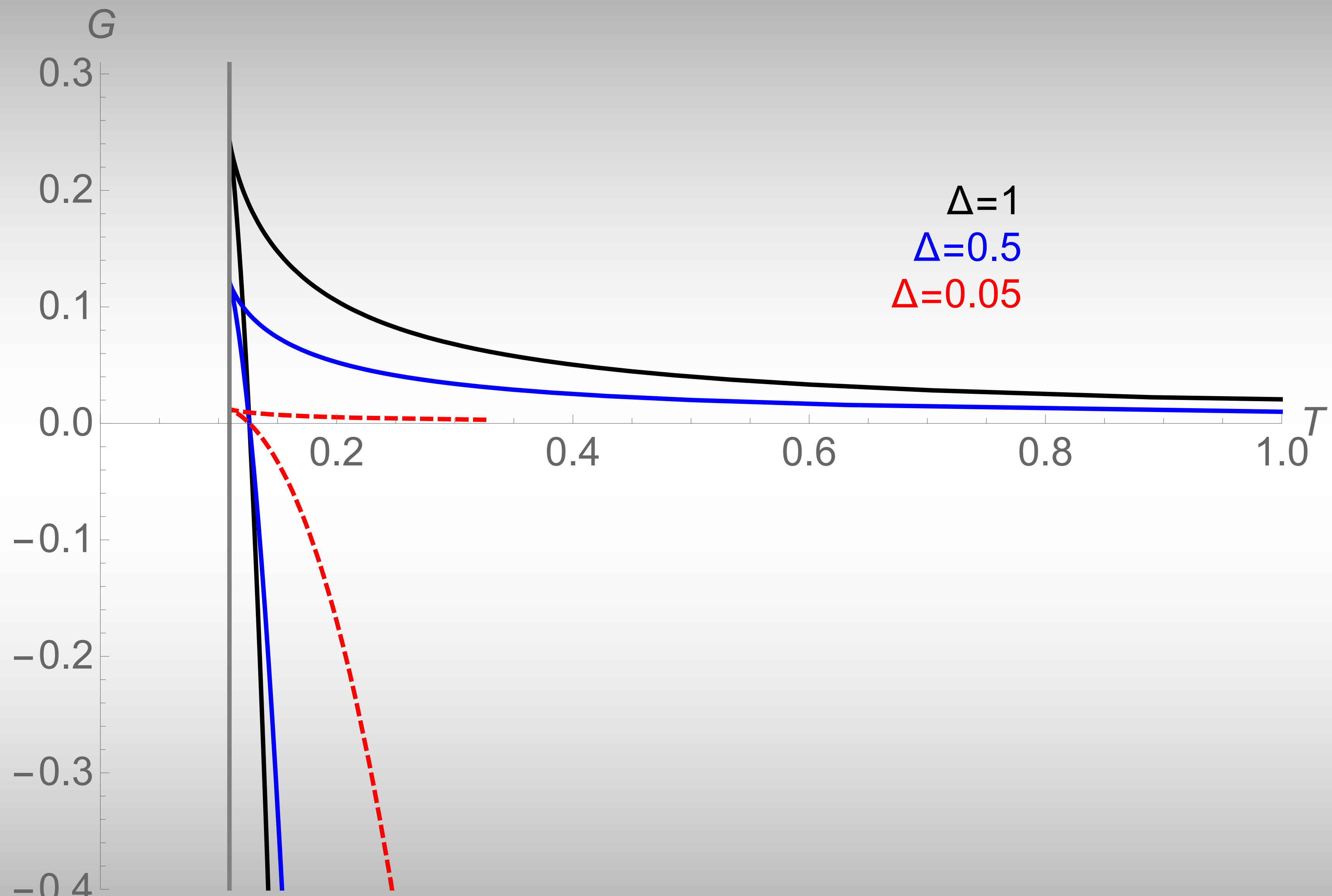
RG & Scoins, 1904.09660

Accelerating Chemistry

To explore how the thermodynamics depend on the deficits, first let C=0, and consider the uncharged nonrotating black hole. It looks like Δ is almost irrelevant, but consider the Gibbs free energy: G = M-TS

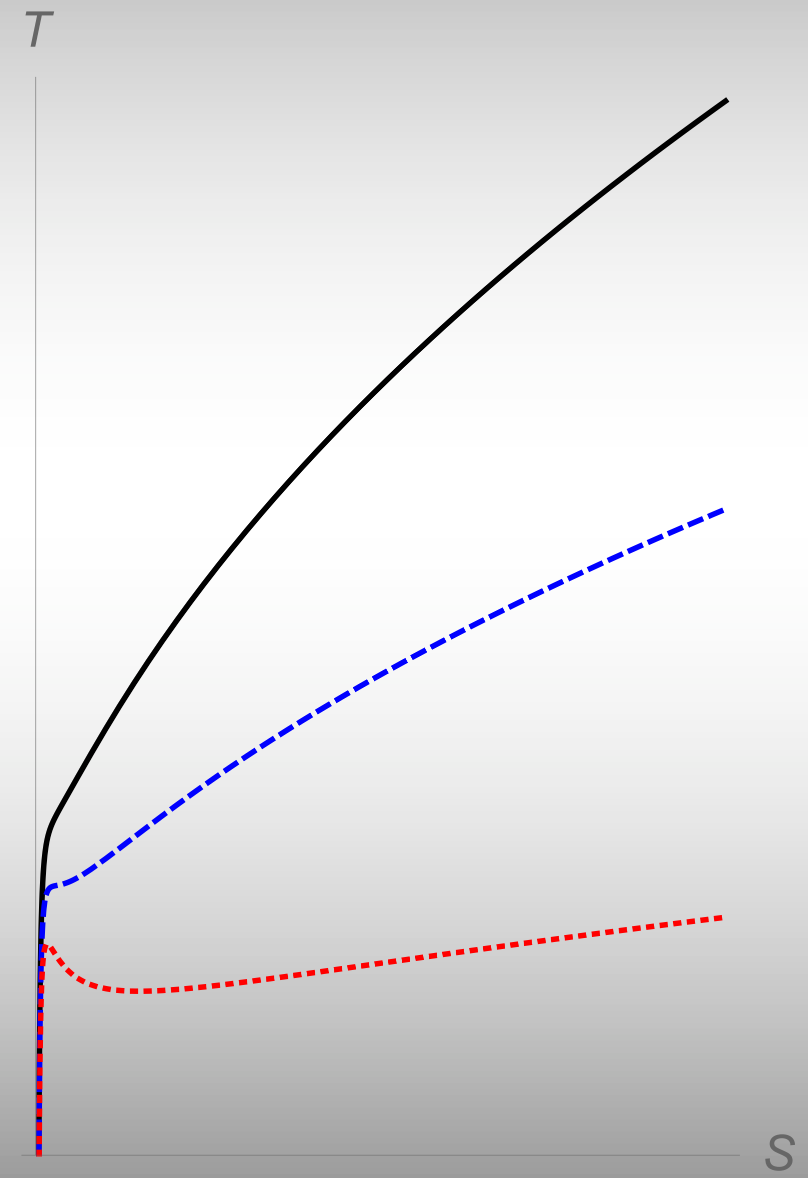
$$G = \frac{\Delta S}{8\pi M}\left(1 + \frac{8PS}{3\Delta}\right)\left(1 - \frac{8PS}{3\Delta}\right)$$

Δ decreases the free energy – though the HP transition remains at the same T.

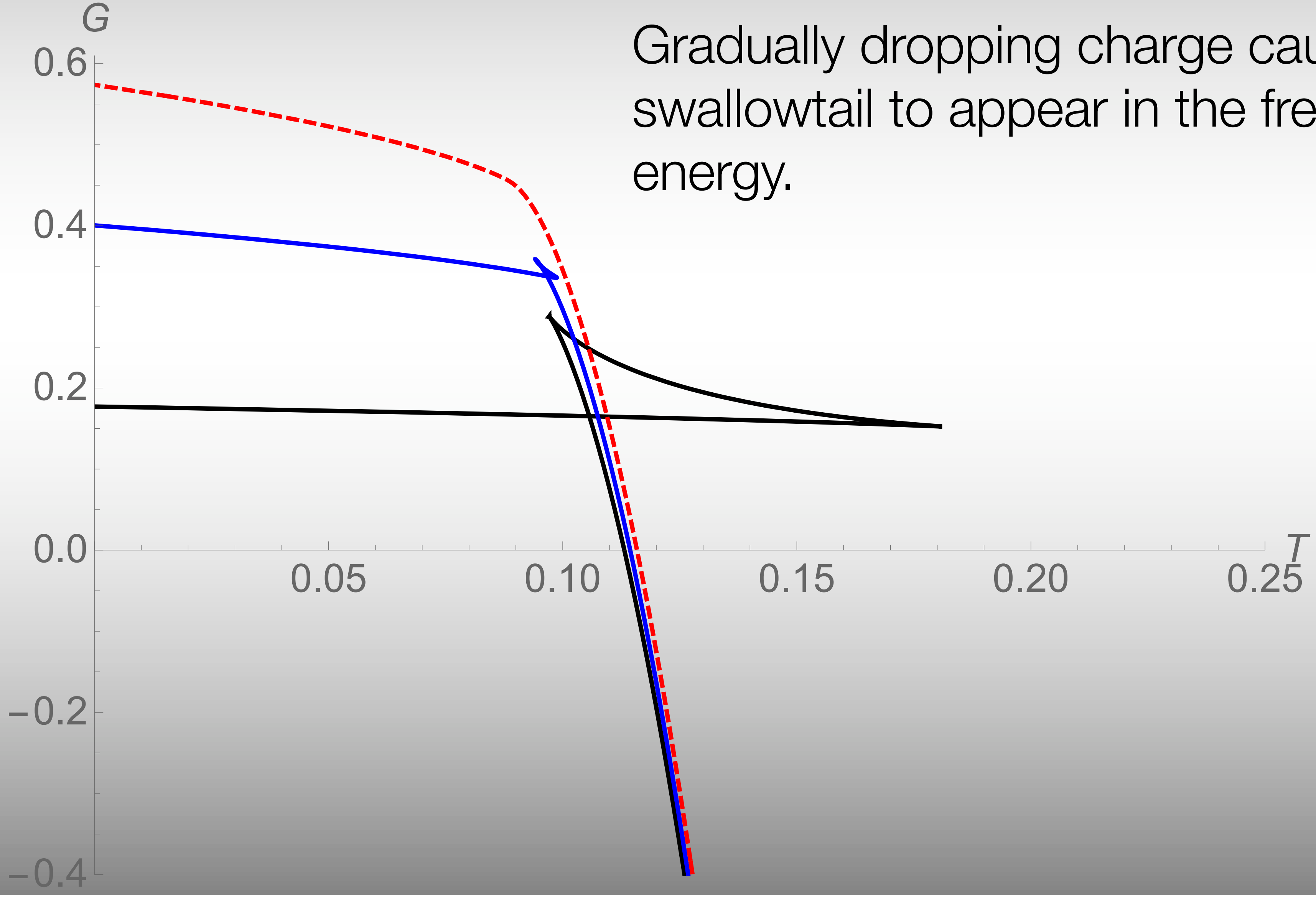

G
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
0.2
0.4
0.6
0.8
1.0
T
Δ=1
Δ=0.5
Δ=0.05

Swallowtails

With charge or rotation, new critical phenomena appear. The temperature can now have two turning points with entropy, if the pressure is low enough relative to charge.



Swallowtails



Gradually dropping charge causes a swallowtail to appear in the free energy.

Snapped Swallowtails

But now something interesting happens with acceleration! Let J=0, and shorten expressions by writing:

$$x = \frac{8PS}{3\Delta}\,, \qquad \frac{\pi Q^2}{\Delta S} = q\frac{C^2}{x}$$

Then we can factorise the expression for M:

$$M^2 = \frac{\Delta S}{4\pi}\left[\left(1 + x - q_+\frac{C^2}{x}\right)\left(1 + x - q_-\frac{C^2}{x}\right)\right]$$

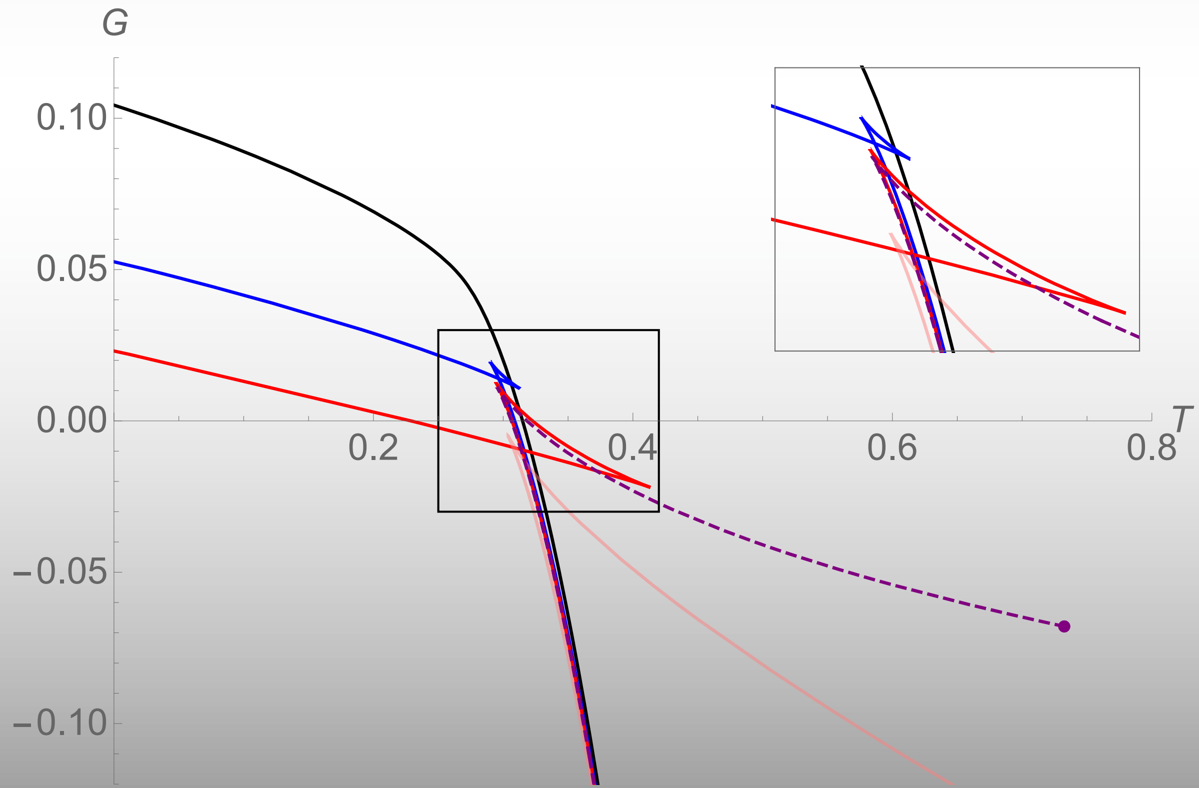
Where $$q_\pm = 2 - q \pm 2\sqrt{1-q}$$

If q<1, these roots are real, and the enthalpy can vanish!

Abbasvandi et al. 1812.00384

Snapped Swallowtails

q relates the magnitude of the charge to the ratio of the deficits and pressure, and as pressure drops, q drops. The swallowtail forms, then snaps at a critical Q.



RG & Scoins

A Theorem: Reverse Isoperimetric Inequality

The Isoperimetric Inequality in Mathematics says that the minimal boundary length enclosing a given area is a circle (or suitable higher dimensional generalisation). This is a problem if true for thermodynamic volume and black holes, since it would say that a round black hole would *minimize* area for a given volume – but entropy should be maximized! Cvetic et al conjectured that black hole satisfied a reverse of this mathematical inequality and demonstrated its validity for known examples.

Reverse Isoperimetric Inequality

Focus on uncharged case:

$$M^2 = \frac{\Delta S}{4\pi}(1+x)\left[1+x-4\frac{C^2}{x}\right]$$

$$V = \frac{2S^2}{3\pi M}\left[1+x+\frac{C^2}{x^2}\right]$$

& combine the expressions

$$\frac{4\pi M^2}{\Delta S} = \left(\frac{3\pi MV}{2S^2} - \frac{2C^2}{x^2}\right)^2 - 4(1+x)\frac{C^2}{x}$$

New Reverse Isoperimetric Inequality

Now can manipulate

$$M^2\left(\frac{3V}{4\pi}\right)^2\left(\frac{\pi}{S}\right)^4 \geq \left(\frac{3\pi MV}{4S^2}-\frac{C^2}{x^2}\right)^2 \geq \frac{\pi M^2}{\Delta S}$$

into a new inequality appropriate for conical deficit black holes:

$$\left(\frac{3V}{4\pi}\right)^2 \geq \frac{1}{\Delta}\left(\frac{\mathcal{A}}{4\pi}\right)^3$$

"C" IN 3

We can look for an exact solution in 3D with the same type of structure:

$$ds^2 = \frac{1}{A^2(x-y)^2}\left[P(y)d\tau^2 - \frac{dy^2}{P(y)} - \frac{dx^2}{Q(x)}\right]$$

With general solution: $Q(x) = c + bx + ax^2\,, \qquad P(y) = \frac{1}{A^2\ell^2} - Q(y)$

Which, after coordinate rescaling/shifts reduces to:

Class	$Q(x)$	$P(y)$	Maximal range of x
I	$1-x^2$	$\frac{1}{A^2\ell^2} + (y^2-1)$	$\lvert x\rvert < 1$
II	x^2-1	$\frac{1}{A^2\ell^2} + (1-y^2)$	$x > 1$ or $x < -1$
III	$1+x^2$	$\frac{1}{A^2\ell^2} - (1+y^2)$	$\mathbb{R}$

Arenas-Henriquez, RG, Scoins: 2202:08823; Anber 0809:2789; Astorino 1101.2616

Accelerating Particle

Take each in turn. The first class looks very similar to the 4D C-metric (r=-1/Ay, t = $\alpha\tau$/A, x = cos(ϕ/K))

$$ds^2 = \frac{1}{\left[1 + Ar\cos\left(\phi/K\right)\right]^2}\left[f(r)\frac{dt^2}{\alpha^2} - \frac{dr^2}{f(r)} - r^2\frac{d\phi^2}{K^2}\right]$$

$$f(r) = 1 + (1 - A^2\ell^2)\frac{r^2}{\ell^2}$$

Slow Acceleration $A\ell < 1$ No horizon

Rapid Acceleration $A\ell > 1$ Acc. horizon

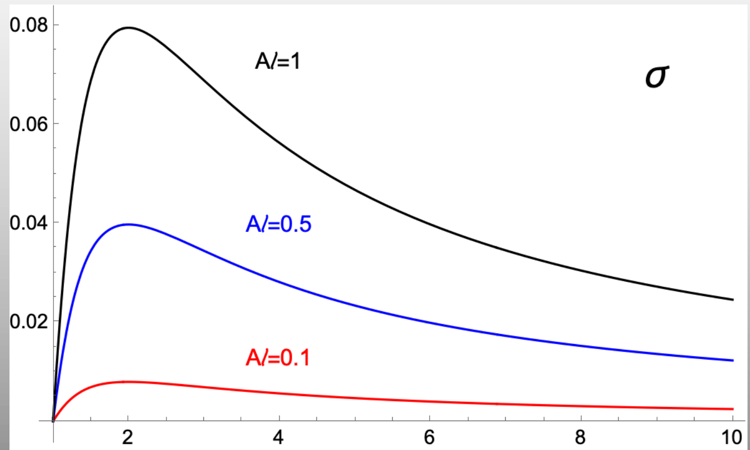
Slow Acceleration

The presence of K now indicates both a conical deficit (the particle) and a *domain wall* at $\phi = \pm\pi$, i.e. codimension 1 defect. The conical deficit at r=0 has a natural mass:

$$m_c = \frac{1}{4}\left(1 - \frac{1}{K}\right)$$

Because of the nonzero extrinsic curvature along $\phi = \pm\,\pi$, (thanks to A) there is a wall of tension

$$\sigma = \frac{A}{4\pi} \sin\left(\frac{\pi}{K}\right)$$

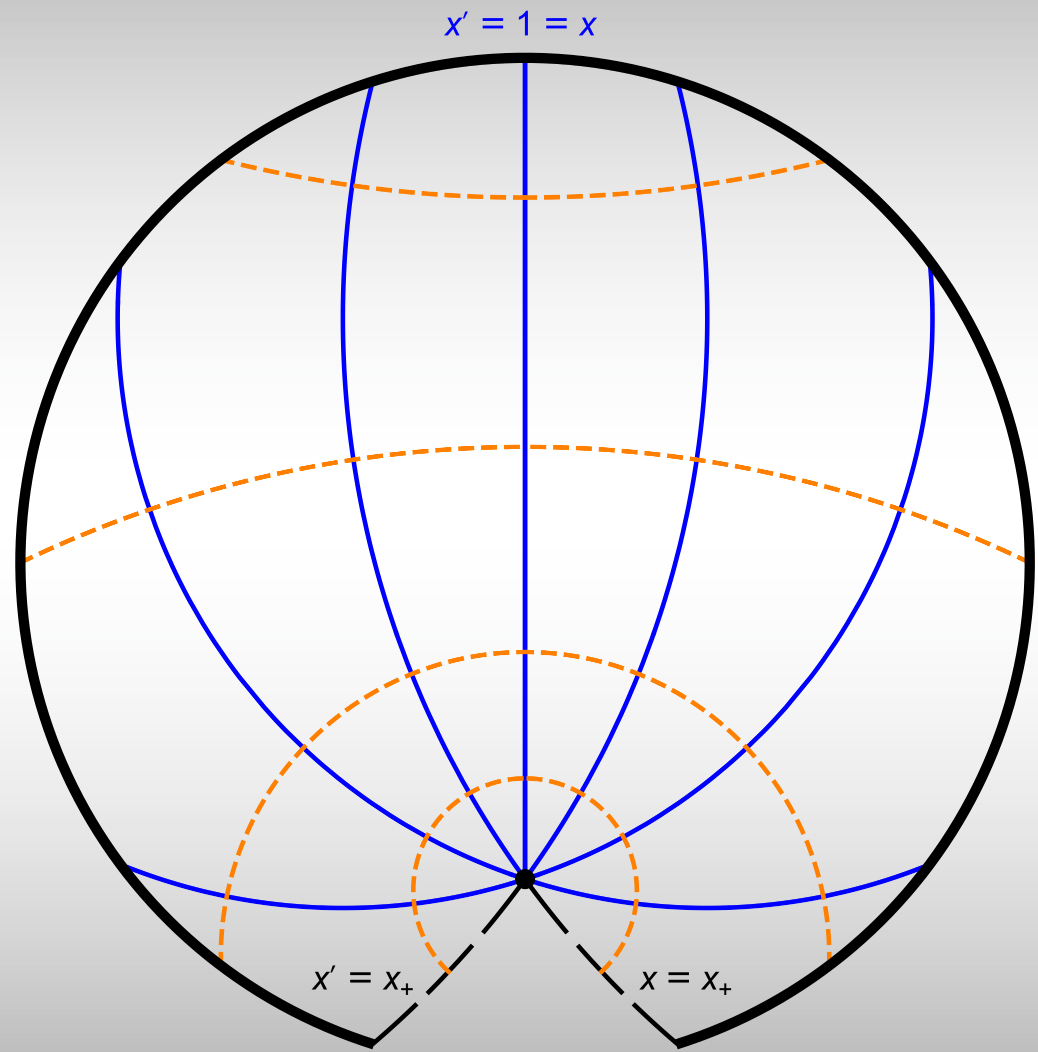


Slow Acceleration

We can do the same coord transformation as in the 4D slow acceleration case, to get the same sort of picture:

$$R_0 = \frac{A\ell^2}{\alpha}$$

A determines the displacement from origin, and x_+ both the particle "mass" and wall tension.



Particle Mass?

Can follow the same Fefferman-Graham prescription as for 4D, giving the expected boundary metric:

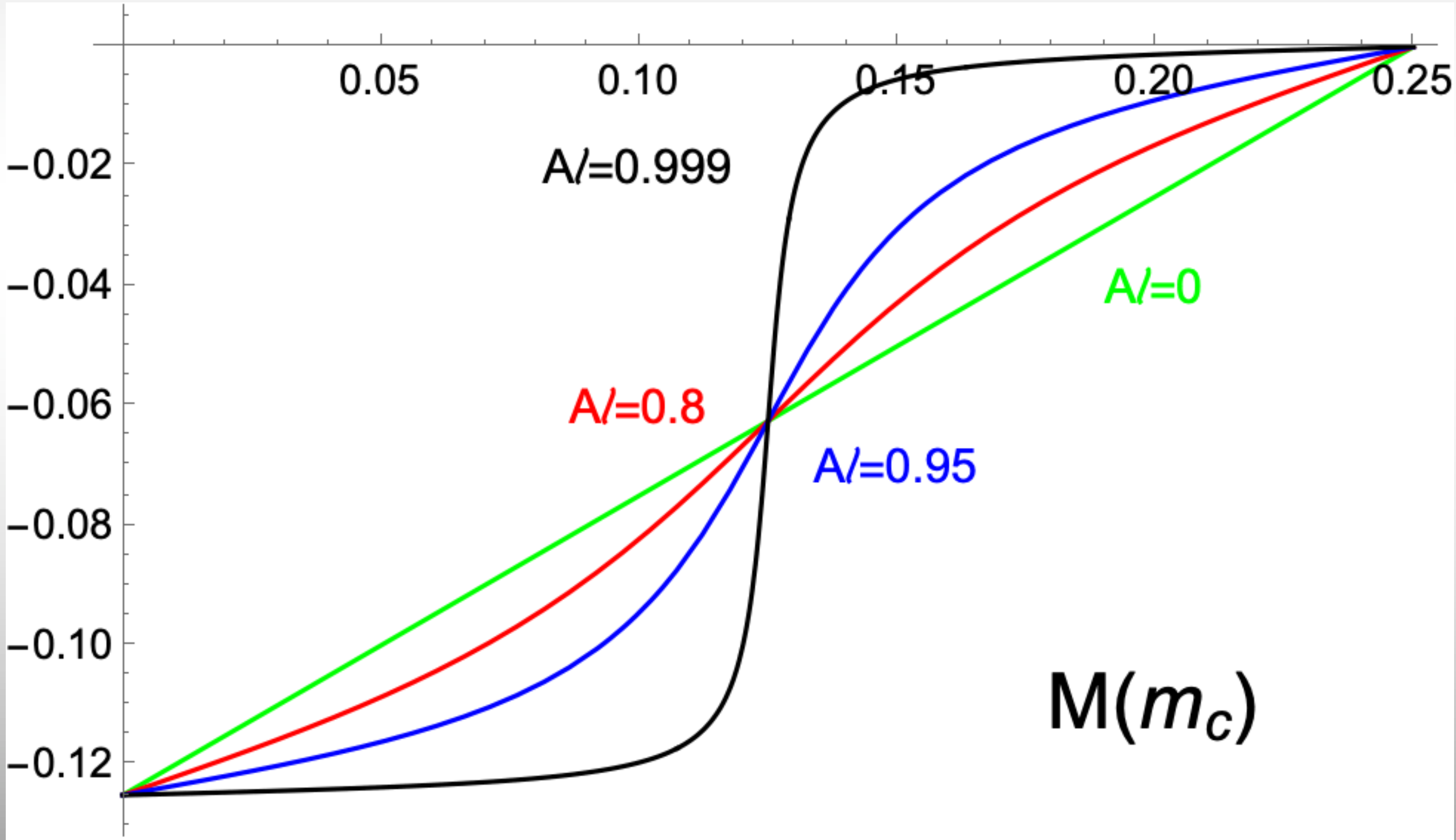
$$\gamma_0 = \frac{\omega(\xi)^2}{A^2}\left[d\tau^2 - A^2\ell^2\frac{d\xi^2}{1-\xi^2}\right]$$

And, after setting alpha to the same value as 4D, the mass:

$$M = -\frac{1}{8\pi}\left(\frac{\pi}{2} - \arctan\left[\frac{\cot\left(\frac{\pi}{K}\right)}{\sqrt{1-A^2\ell^2}}\right]\right)$$

Holographic Mass

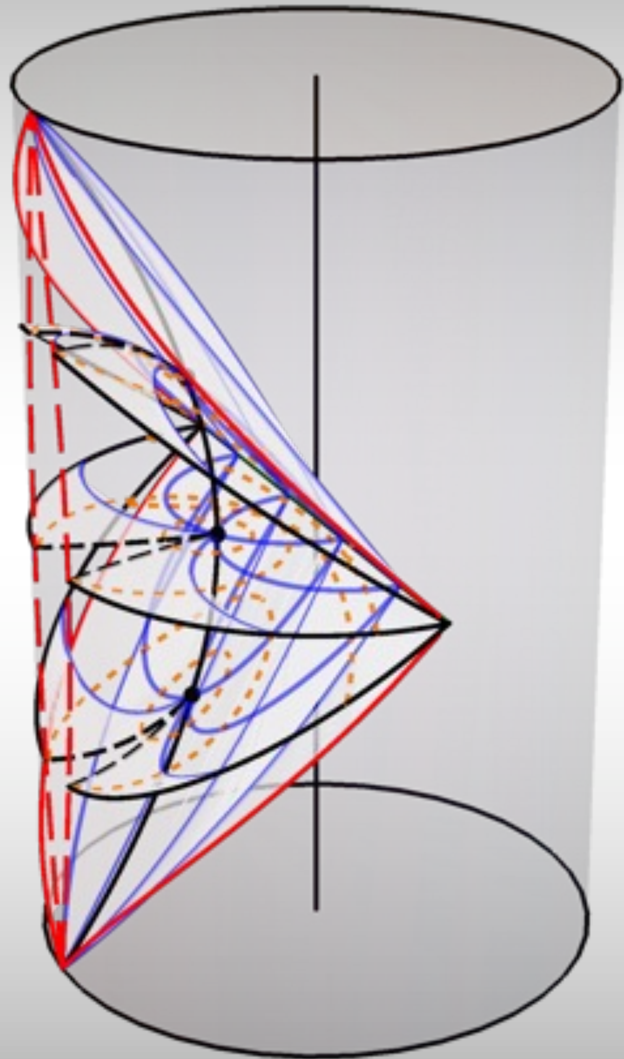
Compare to "particle" mass from conical deficit:

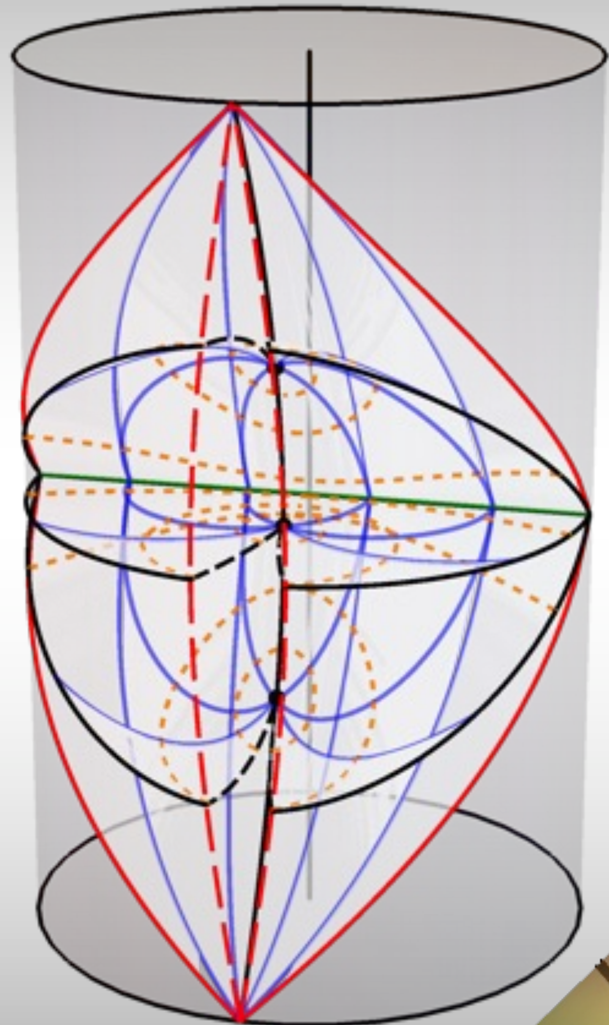


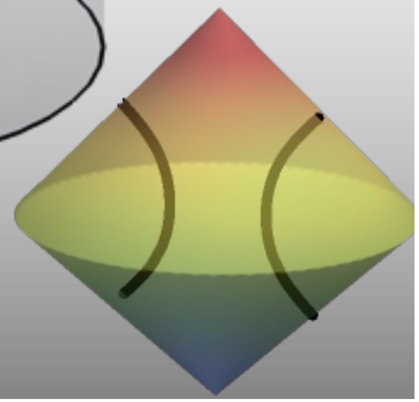
New Solutions?

Although these have been derived as “new” solutions, we know that in 3D, gravity does not propagate, so any ”vacuum” solution has to be locally equivalent to AdS. The transformation formulae for the various solutions are quite lengthy, but give an interesting alternative viewpoint, and help with understanding the “BTZ” family of solutions.

Rapidly accelerating light Particle

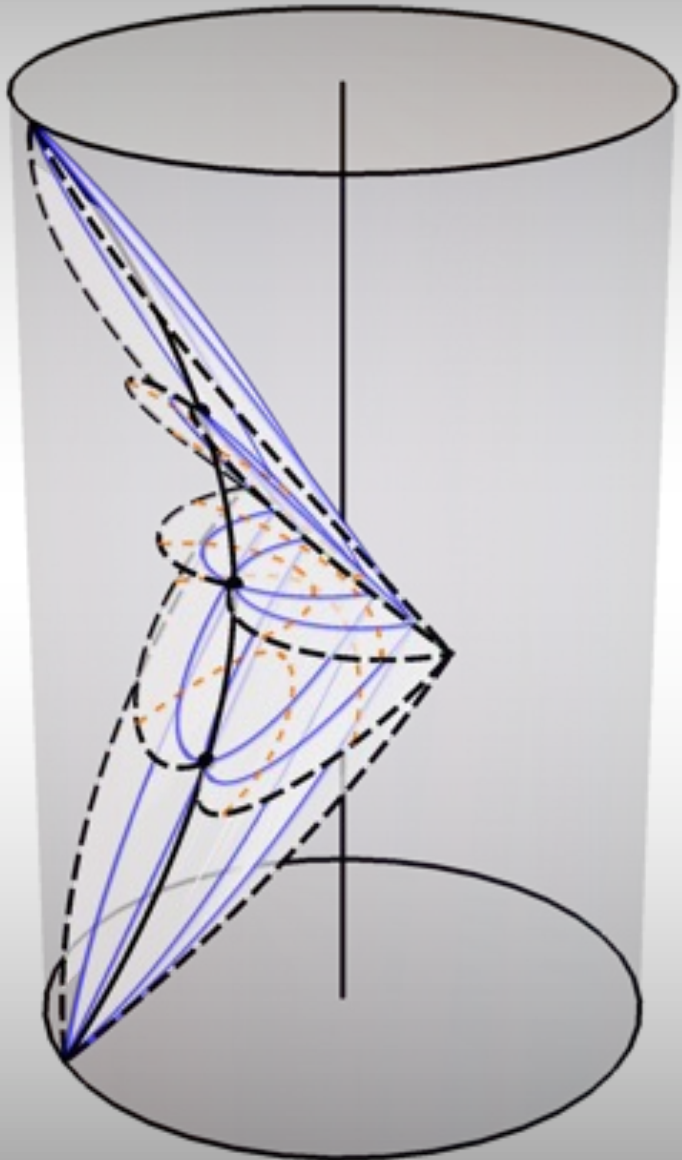


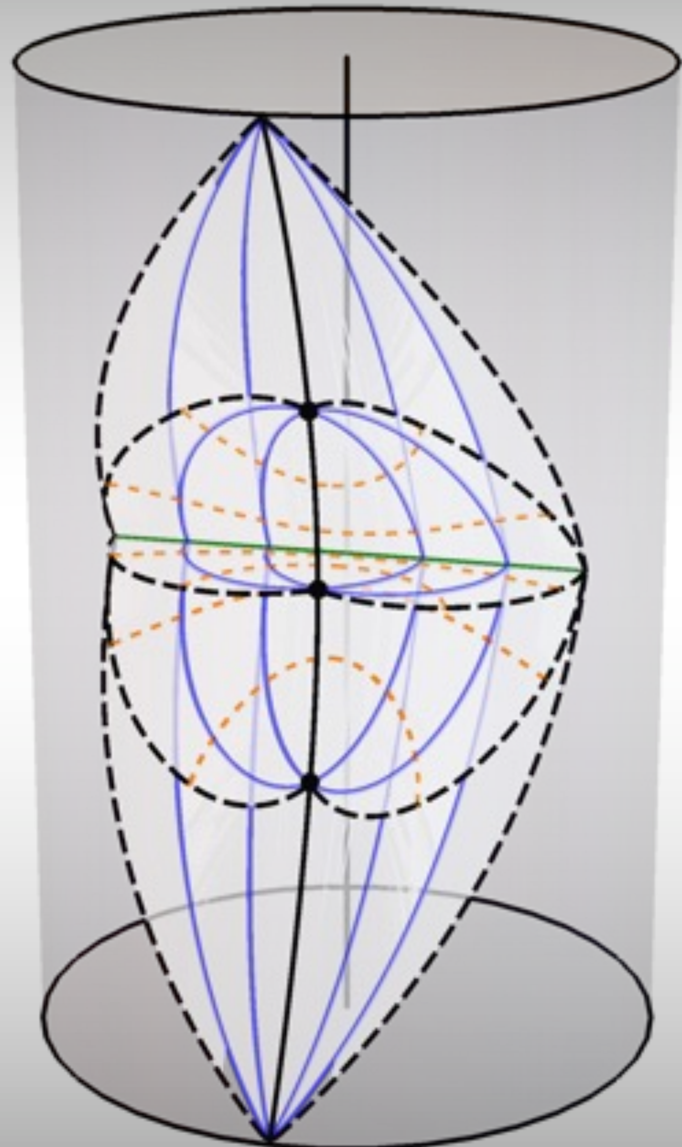




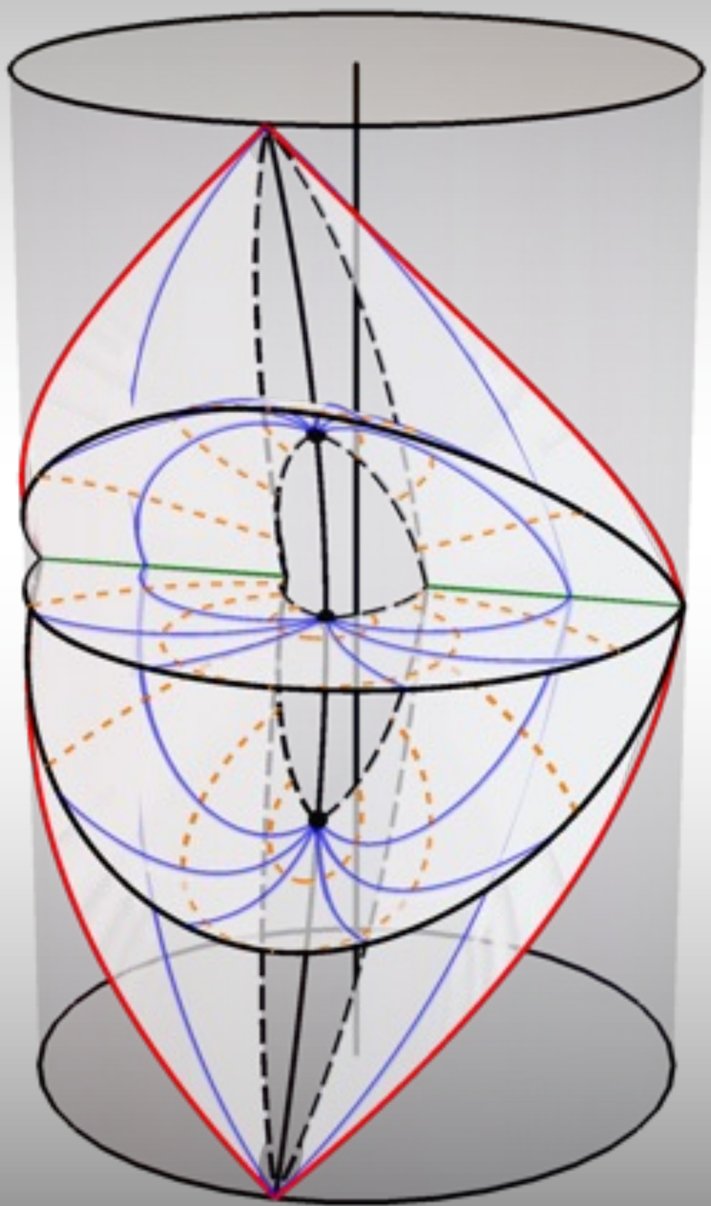
Main difference to 4D: no accelerating partner!

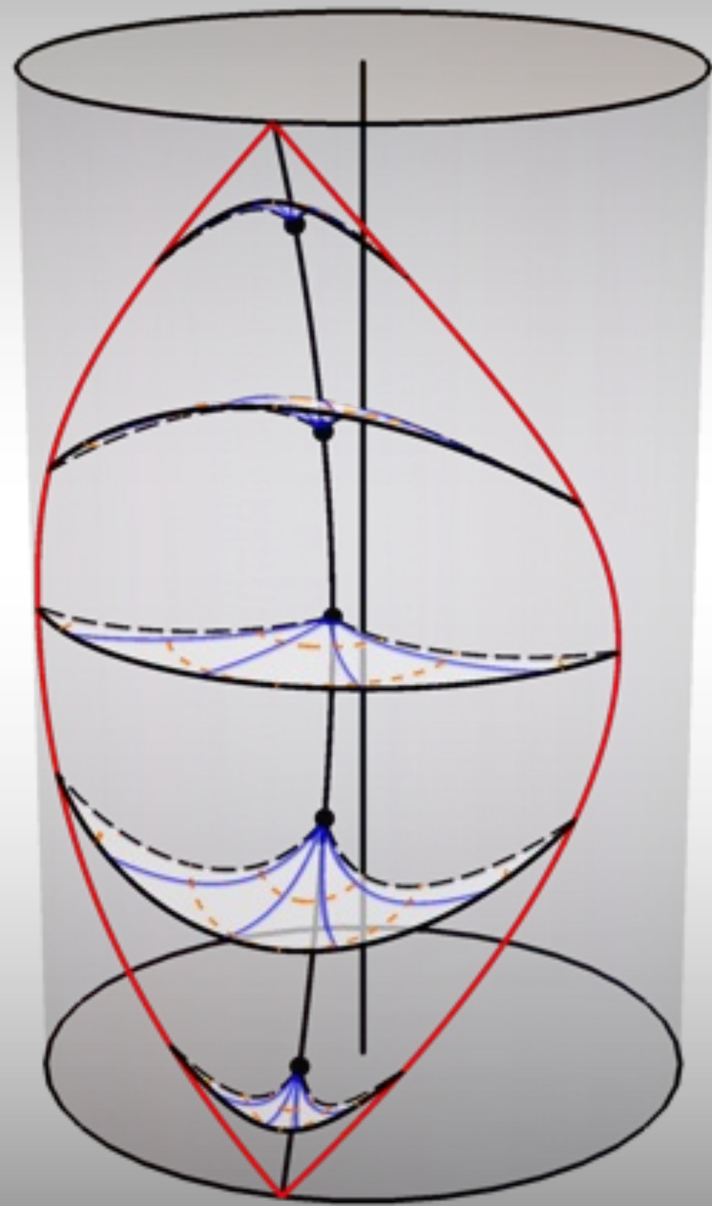
Rapidly accelerating heavy Particle





Acceleration with Struts

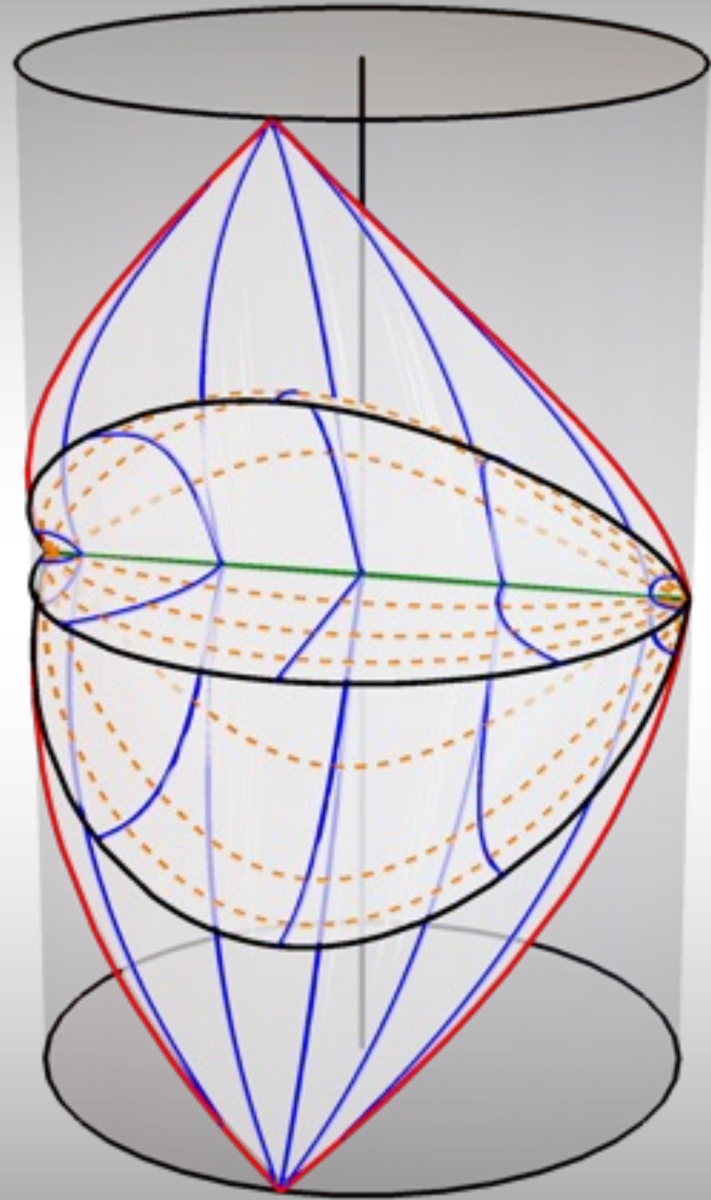




BTZ

Recall the BTZ black hole is an identification of the Rindler wedge:

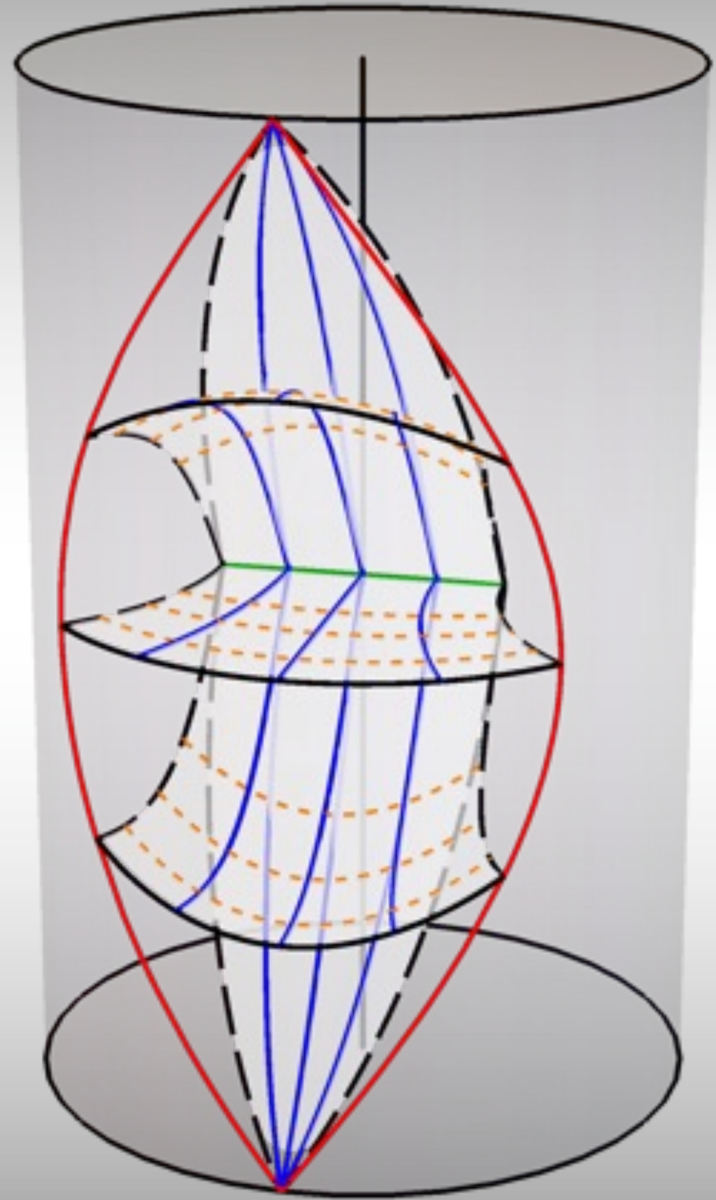
Blue lines are constant ϕ, and have zero extrinsic curvature,



BTZ

Recall the BTZ black hole is an identification of the Rindler wedge:

Blue lines are constant ϕ, and have zero extrinsic curvature, so can cut and paste along ϕ–lines to form the BTZ black hole.



BTZ's as class II

Looking at BTZ from the exact solution perspective:

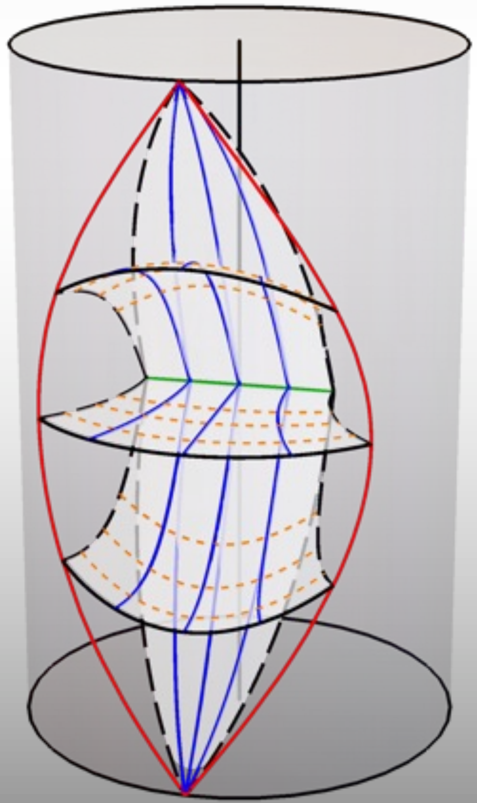
$$ds^2 = \frac{1}{\Omega(r,\psi)^2}\left[F(r)\frac{d\tilde{t}^2}{\alpha^2} - \frac{dr^2}{F(r)} - r^2 d\psi^2\right],$$

$$F(r) = -m^2(1-\mathcal{A}^2 r^2) + \frac{r^2}{\ell^2},$$

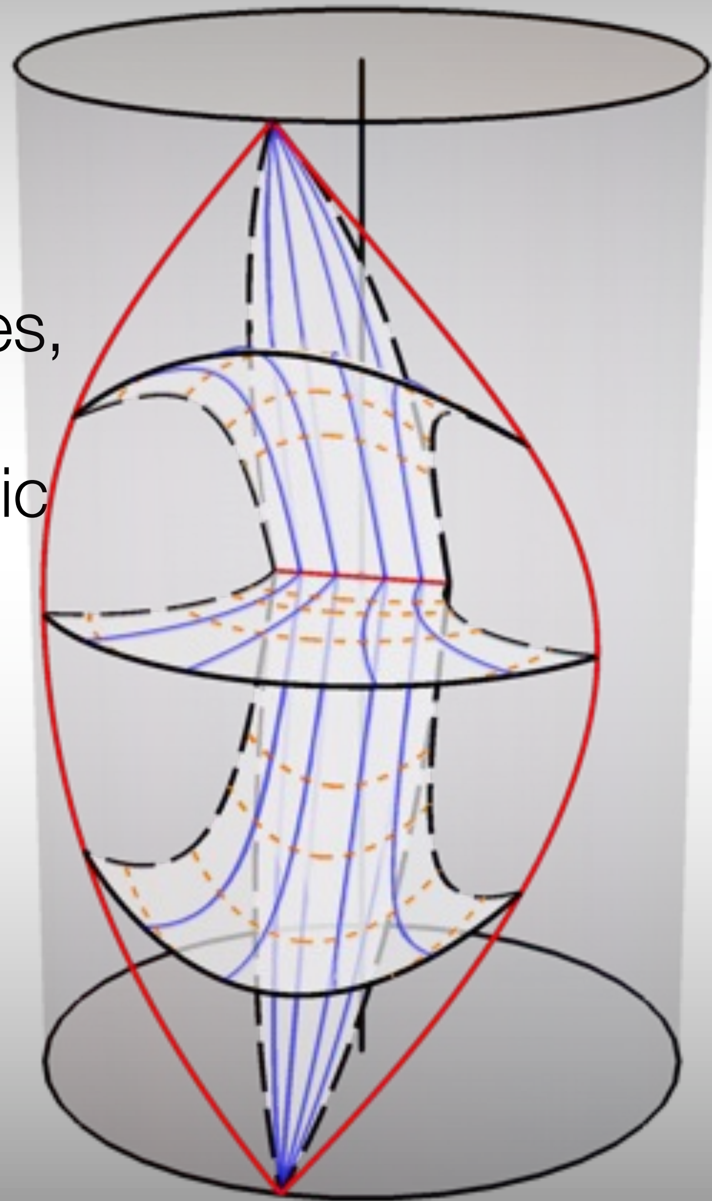
$$\Omega(r,\psi) = 1 + \mathcal{A} r \cosh(m\psi)$$

$$\begin{pmatrix} K = 1/m \\ A = m\mathcal{A} \end{pmatrix}$$

BTZ's

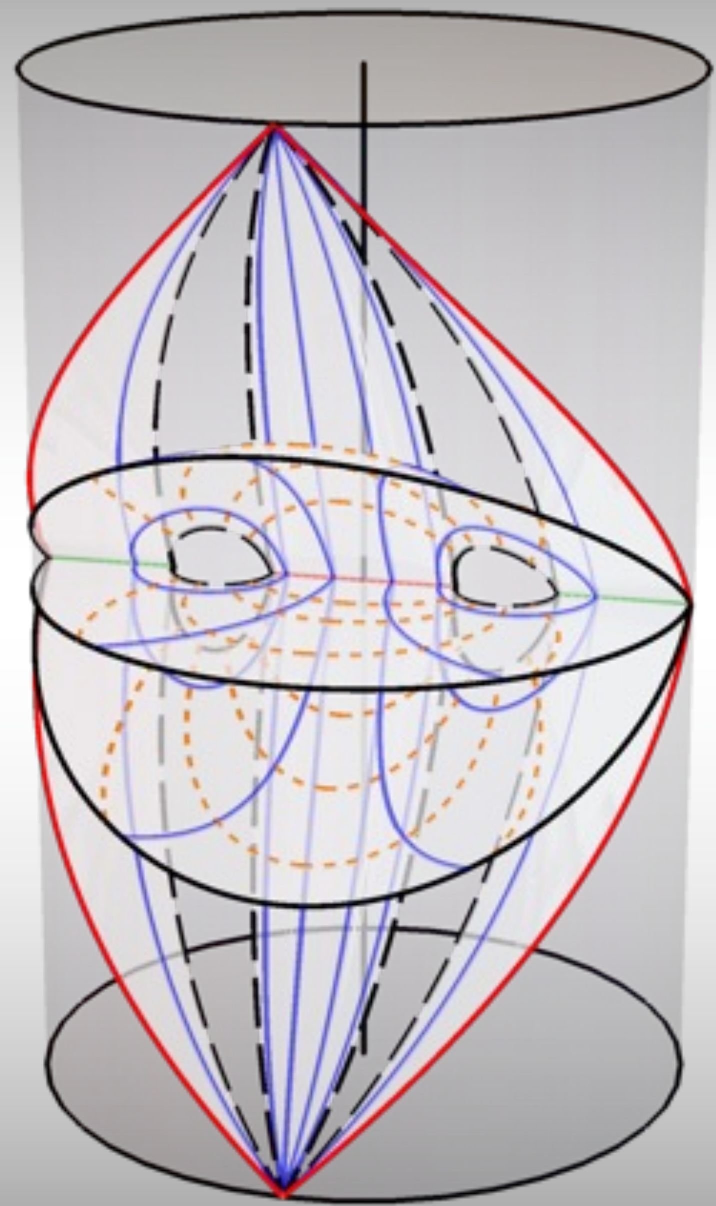


Adding A in the class 2's skews the constant ϕ lines, changing the way AdS is sliced and adding extrinsic curvature to constant ϕ–lines – here is a slightly distorted BTZ (slow acceleration)



Rapid BTZ's

Because the distorted ϕ–lines now wrap back to the Rindler wedge horizon, for some values of ϕ we get an "additional" horizon (different portions of the bulk Rindler horizon).



Novel BTZ

Hiding within class I is a new BTZ-like solution. If Al>1, have a horizon at

$$y_h^2 = 1 - \frac{1}{A^2\ell^2}$$

For the accelerating particle, we usually take y<-y_h with y ~ -1/Ar, but can also have $y \in (y_h, x)$ $\qquad x \in (x_+, 1)$

To make this look more familiar, take

$$\tau = \frac{At}{\alpha}, \qquad y = \frac{1}{A\rho}, \qquad x = \cos(\phi/K)$$

where

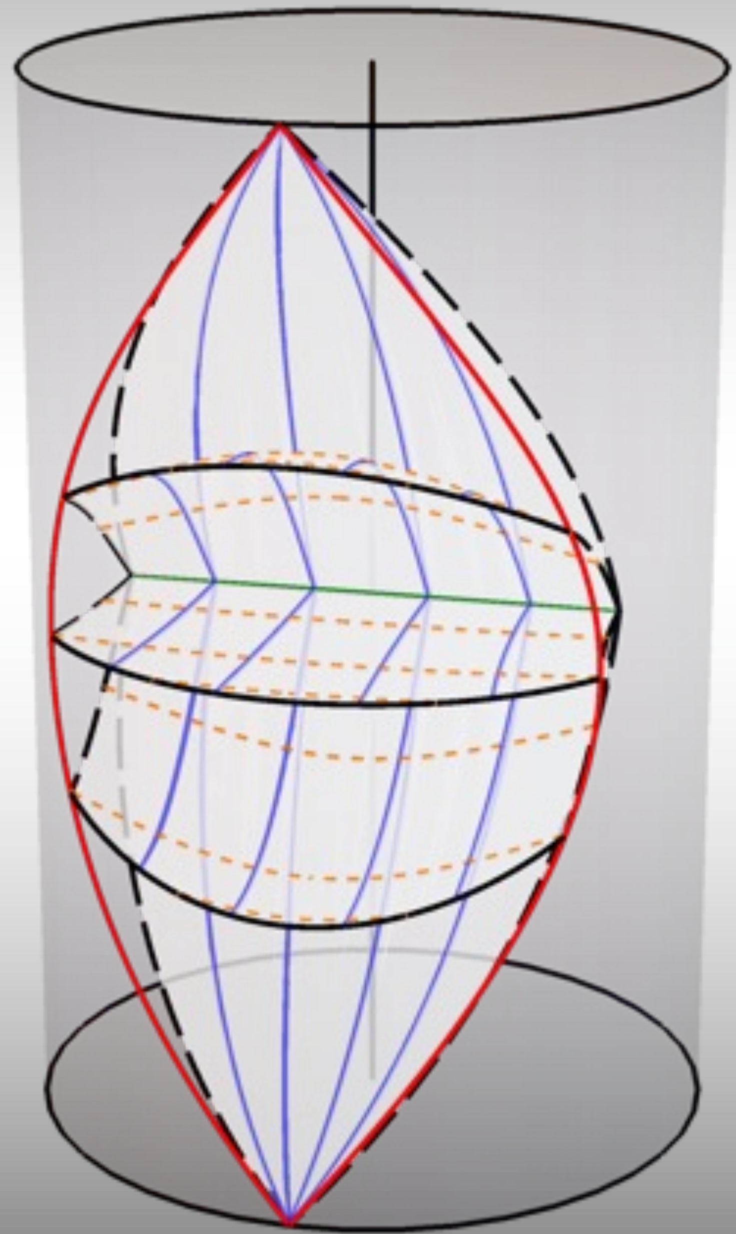
$$K = \pi/\arccos(x_+) > \pi/\arccos(y_h) > 2$$

So that this solution is clearly disconnected from the non-accelerating solutions.

$$ds^2 = \frac{1}{\left[A\rho\cos\left(\frac{\phi}{K}\right) - 1\right]^2}\left(f(\rho)\frac{dt^2}{\alpha^2} - \frac{d\rho^2}{f(\rho)} - \rho^2\frac{d\phi^2}{K^2}\right)$$

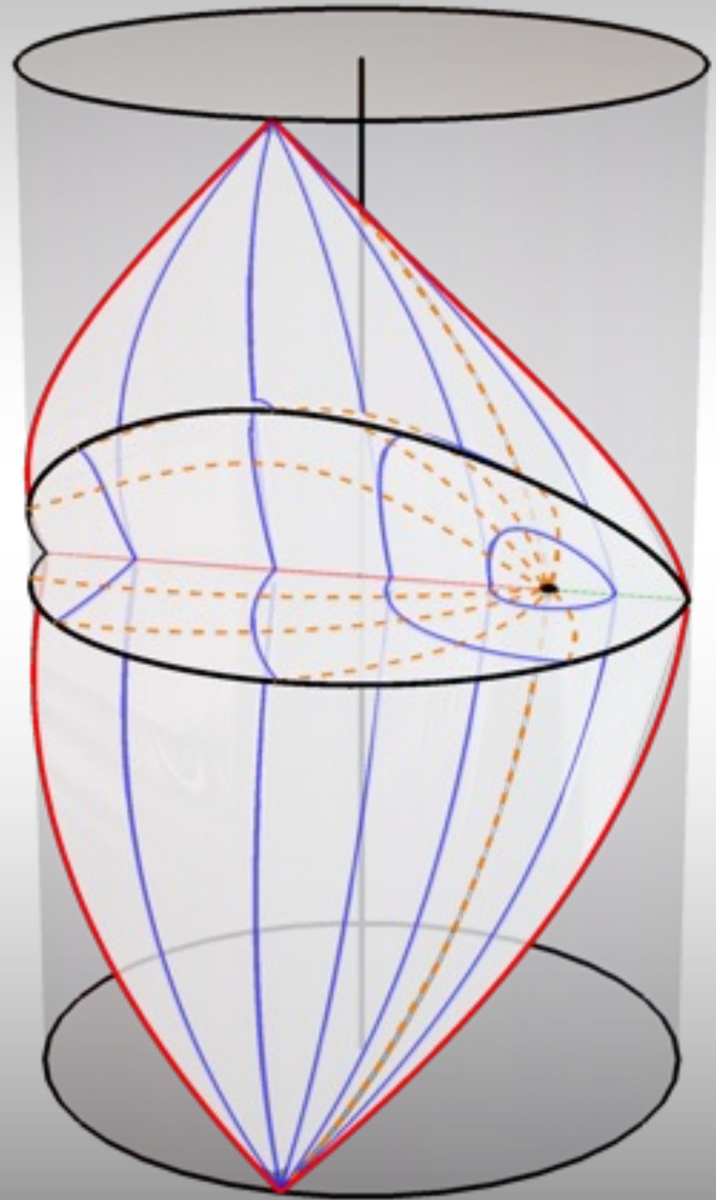
$$f(\rho) = 1 - (A^2\ell^2 - 1)\rho^2/\ell^2$$

Plotting this solution in global coordinates shows a clear parallel with BTZ. This time however, there is no continuous link to the BTZ metric.



Class III - braneworld

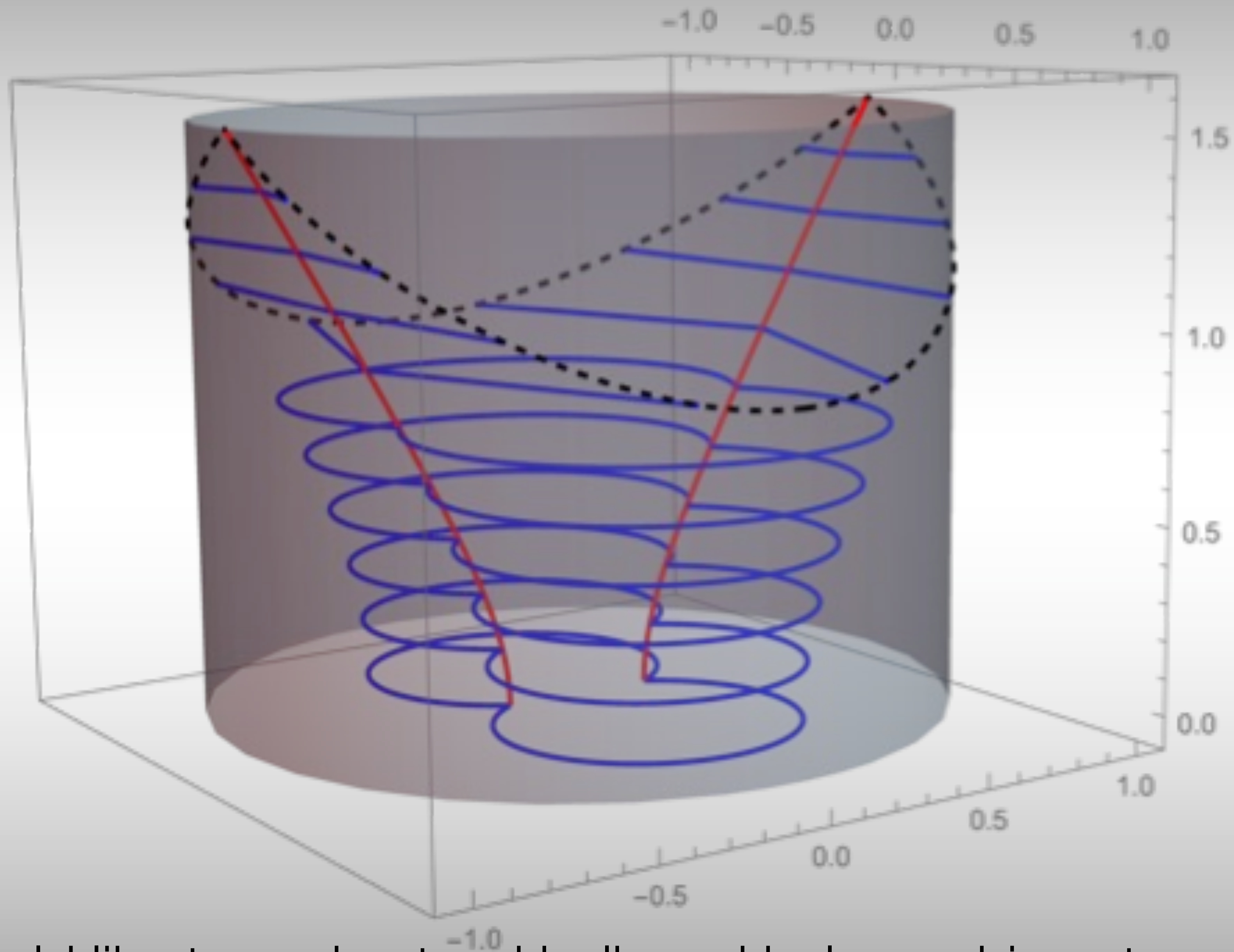
Finally, the class 3 solutions don’t allow for an identification of a single bulk with a wall, instead we take 2 bulk copies a la Randall-Sundrum to form a braneworld.



Recap

- Have shown how to allow for varying tension in thermodynamics of black holes.
- Conjugate variable is *Thermodynamic Length*
- Thermodynamics of accelerating black holes is computable – non-static and non-isolated.
- A key technical point is the normalisation of timelike Killing vector
- Have derived extensive expressions for the TD variables and a new Reverse Isoperimetric Inequality.
- Three dimensions is both familiar and new!

Rapidly accelerating heavy particle – full bulk.



Would like to understand bulk and holographic nature of 3D solutions, as well as thermal back-reaction.