Lecture 8: Gravitational Waves

Christoph Welling
Gravity Reconsidered

• Newton’s Laws of Motion:
  • An object moves in a straight line, at constant speed, unless a force acts on it.
  • \( F = m \times a \)
  • For every force, there is an equal, but opposite, reaction force.

• Newton’s theory of Gravity:
  \[ F = G \times m_1 \times m_2 / d^2 \]
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Euclidean Geometry

Axioms:

1. You can draw a straight line from any point to any other
2. You can extend any finite straight line infinitely long
3. You can draw a circle with any radius around any point
4. All right angles are equal to each other
5. If a straight line crosses 2 other straight lines so that the interior angles on one side are less than 2 right angles, the lines will intersect on that side if extended infinitely long.
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Non-Euclidean Geometry

Nikolai Lobachevsky
Bernhard Riemann
Metrics

If I move a small distance $\Delta x$, how far did I go?

- Euclidean space:
  \[ d = \sqrt{\Delta x_1^2 + \Delta x_2^2 + \Delta x_3^2} \]

- Measure path length by adding up metrics for each little step
- “Straight line”: The path between two points that results in the shortest distance
If I move a small distance \( \Delta x \), how far did I go?

- Euclidean space:
  \[ d = \sqrt{\Delta x_1^2 + \Delta x_2^2 + \Delta x_3^2} \]

- Measure path length by adding up metrics for each little step

- **Geodesic**: The path between two points that results in the shortest distance
General Relativity

- All inertial reference frames are equally valid
- The vacuum speed of light is constant in all inertial reference frames
- It is impossible to distinguish between gravity and acceleration (locally)

Consequence: Masses distort spacetime (i.e. the metric) around them
Refraction
Refraction
Gravitational Lensing
Gravitational Lensing
Gravitational Lensing
Gravitational Lensing
Gravitational Waves
The Hulse-Taylor pulsar

Joseph Taylor

Russell Hulse

periastron
Gravitational Waves from Binary Pulsars

Cumulative shift of periastron time (s)

Year


General Relativity prediction

Nobel Prize
Supermassive Black Holes
Galaxy Mergers
Gravitational Waves in Action
Gravitational Waves in Action
Pulsar Timing Arrays
Pulsar Timing Arrays
Pulsar Timing Arrays

![Graph showing correlation between pulsars and angular separation](image)
Interferometers
The LIGO-Virgo-KAGRA Network
LIGO
LIGO
GW150914: Binary Black Hole Merger
GW150914: Binary Black Hole Merger
Chirp Mass

\[ M = \left( m_1 \cdot m_2 \right)^{\frac{3}{5}} / \left( m_1 + m_2 \right)^{\frac{1}{5}} \]

\[ \frac{df}{dt} = \frac{96}{5} \pi^{\frac{8}{3}} \left( G \cdot M / c^3 \right)^{\frac{5}{3}} f^{\frac{11}{3}} \]
Observed Black Holes
GW 170817

LIGO-Hanford

LIGO-Livingston

Virgo

Frequency (Hz)

Time (seconds)
GRB170817
Multi-messenger Observations of a Binary Neutron Star Merger


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Finding the Source
Follow-up Observations
Broad Emission Lines
Radio Interferometry

Day 75

Day 230
Radio Interferometry
Thank you all for coming!