The 466th Convocation

Address: “A University of Chicago Education and the Pursuit of Truth and Beauty in Mathematics”

By Robert A. Fefferman

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First and foremost, let me express my heartfelt congratulations to the students graduating here today. You have successfully completed the demanding course of study at one of the world’s truly great universities. Our University is tremendously proud to educate a new generation of scholars to take up positions of leadership in a great variety of fundamentally important areas of society. You are those scholars, and we thank you for your hard work and dedication to the life of the mind that we so cherish at the University of Chicago.

As a mathematics professor at the University, it is a pleasure to have this opportunity to share with you some thoughts on the nature of my subject. Unfortunately, it is the case that the average person views this subject as dreary, dull, and technical, enjoyed by a few strange people, and having no real connection to society as a whole. This point of view is even shared by a considerable number of extremely accomplished and otherwise well-educated people. This is due to a number of factors: First, mathematics is indeed somewhat technical, and in order to understand much of it, one must in effect learn an entirely new language. This makes it more difficult to communicate to the non-expert than many other fields. Second, and probably even more important, the quality of pre-collegiate mathematics education in this country is, on average, very, very poor. Early on, children are often taught the subject by teachers who have no specialized training even in the most basic arithmetic, and who have a definite aversion to the material they are supposed to teach. These teachers then efficiently pass on their dislike of the subject to the students.

Furthermore, the study of mathematics is cumulative, so that when one teacher is ineffective and the student has no understanding of foundational material, this may well ruin his or her mathematical education for many years to come, even if there are excellent teachers in those years. Lastly, I think mathematicians are to some extent to blame for the bad reputation of their subject. Some time ago, a famous mathematician who had just been awarded a very prestigious prize was interviewed on television and asked whether it would be possible for him to explain, roughly and in a few sentences, what his work was all about. He immediately blurted out: "No, it was not possible," and the interview was over. It is important for the future of mathematical research that the average person, especially the average educated person who is in a position of leadership and influence, understand more fully the true nature of mathematics, its beauty and absolutely fundamental importance.

Of course with this audience, at a University where intellectual breadth and multidisciplinary research are particularly valued, pointing out the need to appreciate the nature of mathematics is quite obviously preaching to the choir. However, please allow me to relate a few observations and anecdotes that illustrate the true nature of mathematics, because I hope that they will very much capture, at the same time, the spirit and set of guiding principles of this University.

First of all, the pursuit of truth in mathematics is exact and uncompromising. 1 + 1 = 2 and it cannot = 3. There is a certain precision in the subject that means that everyone knows when you are right and when you are wrong, and there is usually no middle ground. More often than not, the important problems are very simple to state and have been worked on by a number of outstanding mathematicians over the years without success, and it takes a certain courage to attack these problems knowing this. Similarly, I would think everyone would agree that our University pursues excellence in an uncompromising manner. While others water down courses to cater to their student bodies, our University offers extremely rigorous courses to students who are not satisfied with anything less. While many other universities have admitted students according to a varying recipe involving many criteria of questionable relevance, ours has never taken anything into account except what really ought to count—the applicant's scholarship and intellectual potential. And, in a sense, it takes a certain amount of courage to go to a school like this one, where the standards are so high and where the culture is so absolutely committed to the life of the mind.

Another striking feature of excellent mathematics is the presence of beauty and especially beauty that is somehow connected with a large element of surprise. In order to explain the meaning of this, let me mention a remarkable University of Chicago story that illustrates beauty and surprise in mathematics as well as any that I know. As everyone is aware, during World War II a number of extremely distinguished scientists emigrated from Europe to the United States, and many of these came to the University of Chicago. Among them was Antoni Zygmund, one of the greatest mathematicians of his time.

Zygmund was a specialist in a branch of mathematics known as Fourier Analysis, in which complicated mathematical objects are broken down as a superposition of simple ones. Its methods involve generalizations of calculus—full of integrals, derivatives, and their more sophisticated extensions. Zygmund, who arrived in the United States in 1940, came to the University of Chicago in 1947. He proceeded to establish the most famous school of mathematical analysis in the United States here in the years that followed. The Chicago School of Analysis, as it was called, featured a remarkable number of brilliant graduate students who were students of Zygmund, or in some cases, students of his students. One of these was a man by the name of Paul Cohen.

Mr. Cohen was a first-rate analyst who, through a friend, became interested in a very famous problem from an entirely different area of mathematics, namely mathematical logic. This was the so-called Continuum Hypothesis. The Continuum Hypothesis related to the simple idea of comparing the size of two sets of objects. If both sets are finite one simply counts how many objects are in each set, and the one with the higher number is the larger set. But in the case where both sets are infinite, it was only in the late nineteenth century that mathematicians discovered a precise way of comparing the size of the two sets. According to this beautiful theory, two infinite sets can actually have different sizes—in other words, there are different orders of infinity. For example, it is a very simple and fundamental result that there are not as many counting numbers, 1, 2, 3, 4, etc., as there are points on a line. The Continuum Hypothesis is the hypothesis that there are no sets with a size between these two sets. What Paul Cohen proved to settle this problem truly shocked the entire world of mathematics: He proved that no such set intermediate in size could be found, and at the same time he showed that it was impossible to prove that no such set exists. In other words, Cohen proved rigorously that the Continuum Hypothesis simply could not be settled one way or the other. It is a tremendous understatement to call this a surprise.

In a field which was always regarded as clear-cut, where every question had a definite answer, this was an intellectual lightning bolt, one that has taken its place in the history of human thought.

That ideas or truths can spring out at you in the most remarkable way as they did in Paul Cohen's work is one of the most appealing characteristics of mathematics, and at the same time of a great education. But there is another somewhat different type of surprise that is just as important. This occurs when a piece of mathematics that is studied for its own sake all of a sudden finds application in a completely unexpected way. Let me tell one more anecdote, also involving Antoni Zygmund, which illustrates this. Although Zygmund was quite productive into his mid-seventies, and this was often a source of inspiration to his younger colleagues, there came a time in the final part of his life when he slowed down in a way that suggested that something was wrong. I remember well the concern that many of us felt over this.

No one helped him more during this difficult period than Izaak and Pera Wirszup, two distinguished members of the University community. On one occasion, Pera took him to a doctor who was to perform brain scans in order to see what was responsible for his deteriorating condition. At this test, which revealed his Alzheimer's disease, the attending physician at the University of Chicago Hospitals who knew that Zygmund was a University mathematics professor, asked Pera for further details as to what kind of mathematical contributions he had made. Pera, whose specialty is far from mathematics, could only answer that she was unsure of the details, but she knew that he was considered the father of modern Fourier Analysis.

Immediately, the doctor recognized the field and its significance, and called a number of his colleagues over. As they gathered around Zygmund, the doctor said, "Without this man's field of mathematics, none of these instruments would be here today." Certainly the early pioneers of Fourier Analysis never dreamed of the idea of the CAT scan, but the methods of Fourier simply were found to be the relevant ones for this application.

There are many other examples of this in Zygmund's own work. For example, there is an idea which is a product of the so-called Calderon-Zygmund theory, the idea of wavelets, that is currently used to greatly improve upon previous methods of image processing. Thanks to wavelets, CAT scans and MRIs may take considerably less time and the images will be a great deal clearer, allowing for much more accurate diagnosis of various serious disorders. And wavelets have many other uses, from medicine to the way the FBI currently processes fingerprints. And this broad applicability is a feature that cannot be overstated when it comes to analyzing the importance of mathematics to society.

Similarly, there is also this type of unforeseen application present in the education that our students receive here. We are certainly not a technical school, and yet by a deep investigation of basic human knowledge, whether it takes place in the College, or in the divisions, our students are able to find the most remarkable applications of their education in a wide assortment of fields. Some of them will be engaged in basic research which will directly make use of their Ph.D. work. Others will pursue entirely different areas seemingly unrelated to the exact subject content of their courses. But there will be many exciting applications here as well because most importantly, our students have learned to think effectively, and that is something that will serve them well as long as they live in whatever they choose to do.

I would like to conclude the way that I began—by congratulating our graduates, and sharing with them the enthusiasm they have felt over the beauty and wonder that was so much a part of their University of Chicago experience. It is most enjoyable to try to imagine what marvelous use the graduates here will make tomorrow of their education we celebrate today. May all of you enjoy the wonderful surprises and accomplishments that await you as you pursue your chosen field, and may you use these to enrich the lives of us all!

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