



THE UNIVERSITY OF
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Computational and Applied Mathematics
&
Statistics Student Seminar

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Bessel-Powered Dirichlet-Multinomial Dynamical Systems

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ABSTRACT

A pillar of Bayesian computation is the closed-form conjugate prior: a tractable prior distribution whose posterior under a given likelihood belongs to the same family. We show that reversing the roles of the conjugate prior and its likelihood — that is, using the prior as a likelihood with the likelihood as its prior — is surprisingly tractable in special cases, in particular, for the multinomial and Dirichlet distributions. While the Dirichlet is the conjugate prior of the multinomial, the reverse direction — conditioning on a Dirichlet observation to recover latent multinomial counts — produces an intractable posterior. However, we show that endowing the total count of the multinomial with a Bessel distribution reveals a remarkably tractable posterior marginal. This construction defines the Bessel-Multinomial-Dirichlet (BMD), a new family of probability distributions on the simplex that generalizes the Dirichlet. Composing the BMD over time produces a Markov chain on the simplex whose moments, spectral structure, and mixing behavior are all available in closed form through ratios of Bessel functions. We demonstrate how the interpretable dynamics of the chain lead to tractable and practical inference for dynamic topic models and sparse compositional time series.