THE UNIVERSITY OF CHICAGO

Computational and Applied Mathematics

& Statistics Student Seminar

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Numerical Stability and Tensor Nuclear Norm

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ABSTRACT

Recently, a team from Google's DeepMind discovered more than 14, 000 non-equivalent 49- term decompositions for 4×4 matrix product using AI. When one has so many different algorithms, the question becomes which one to pick? The most natural criteria would be numerical stability — out of the 14,000 candidates, we should pick the stablest one. We will show that there is a straightforward measure of numerical stability consistent with the use of tensor rank to measure speed. We present a notion of bilinear stability, which quantifies the numerical stability of an algorithm. An algorithm for evaluating a bilinear operator $\beta : U \times V \to W$ is a decomposition $\beta = \varphi_1 \otimes \psi_1 \otimes w_1 + \cdots + \varphi_r \otimes \psi_r \otimes w_r$; the growth factor of the algorithm $\|\varphi_1\|_*\|\psi_1\|_*\|w_1\| + \cdots + \|\varphi_r\|_*\|\psi_r\|_*\|w_r\|$ captures the accuracy of the algorithm; and its smallest possible value, i.e., the tensor nuclear norm of β, quantifies the accuracy of a stablest algorithm. To substantiate this notion, we establish a bound for the forward error in terms of the growth factor and present numerical evidence comparing various fast algorithms for matrix and complex multiplications, showing that larger growth factors correlate with less accurate results. Compared to similar studies of numerical stability, bilinear stability is more general, applying to any bilinear operators and not just matrix or complex multiplications; is more simplistic, bounding forward error in terms of a single (growth) factor; and is truly tensorial like bilinear complexity, invariant under any orthogonal change of coordinates. As an aside, we study a new algorithm for computing complex multiplication in terms of real, much like Gauss's, but is optimally fast and stable in that it attains both tensor rank and nuclear norm.