Asymmetric Transport in Continuous Topological Insulators

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ABSTRACT

This thesis concerns continuous models for 2-dimensional topological insulators. Such systems are characterized by asymmetric transport along a 1-dimensional curve representing the interface between two insulating materials. The asymmetric transport is quantified by an interface conductivity. In two distinct settings, we derive tractable analytic formulas for this interface conductivity and provide a large class of perturbations under which it is stable. Our theory applies to models of twisted bilayer graphene, low-energy superconductors, and relativistic electrons (possibly subject to a magnetic field) described by an appropriate Dirac operator, among others.

Our second main focus is numerical approximations of the above systems. We define a modified interface conductivity on a box with periodic boundary conditions, and show that it is stable and converges rapidly to its infinite-space analogue as the size of the box goes to infinity. We illustrate with several examples that one can restrict a topological insulator to a large and discrete torus to obtain accurate numerical evaluations of the conductivity. Numerical techniques that do not require periodic truncation are also implemented and analyzed. We derive a novel integral equation for the time-harmonic Klein-Gordon equation with appropriate jump conditions along a one-dimensional interface. We implement a fast multipole and sweeping-accelerated iterative algorithm for solving the integral equations, and demonstrate numerically the fast convergence of this method. Several numerical examples of solutions and scattering effects illustrate our theory.