THE UNIVERSITY OF CHICAGO | THE COMMITTEE ON COMPUTATIONAL AND APPLIED MATHEMATICS

COLLOQUIUM

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Convergence of Unadjusted Langevin and HMC in High Dimensions: Delocalization of Bias

THURSDAY, December 4th at 4:00 PM
Jones 303, 5747 S. Ellis Ave. Chicago, IL 60637

ABSTRACT

The unadjusted Langevin algorithm is commonly used to sample probability distributions in extremely high-dimensional settings. However, existing analyses of the algorithm for strongly log-concave distributions suggest that, as the dimension d of the problem increases, the number of iterations required to ensure convergence within a desired error in the W2 metric scales in proportion to d or its square root. In this paper, we argue that, despite this poor scaling of the W2 error for the full set of variables, the behavior for a small number of variables can be significantly better: a number of iterations proportional to K, up to logarithmic terms in d, often suffices for the algorithm to converge to within a desired W2 error for all K-marginals. We refer to this effect as delocalization of bias. We show that the delocalization effect does not hold universally and prove its validity for Gaussian distributions and strongly log-concave distributions with certain sparse interactions. Our analysis relies on a novel W2,ℓ∞ metric to measure convergence. A key technical challenge we address is the lack of a one-step contraction property in this metric. Our results cover both the underdamped and overdamped Langevin schemes as well as an unadjusted version of the popular Hybrid (or Hamiltonian) Monte Carlo algorithm.

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