THE UNIVERSITY OF CHICAGO | THE COMMITTEE ON COMPUTATIONAL AND APPLIED MATHEMATICS

SPECIAL COLLOQUIUM

KAIBO HU

Mathematical Institute
University of Oxford

Towards Finite Element Tensor Calculus: from Computational Topological Hydrodynamics to Discrete Geometric Structures

TUESDAY, March 31st at 4:00 PM
Jones 303, 5747 S. Ellis Ave. Chicago, IL 60637

ABSTRACT

Finite Element Exterior Calculus (FEEC) and Finite Element Tensor Calculus (FETC) provide a cohomological framework for the structure-preserving discretization of partial differential equations.

The computation of long-term dynamics of finite-dimensional Hamiltonian systems has inspired the development of symplectic algorithms and geometric numerical integration. The development of FEEC over the past decades now enables the study of the analogous problem for fluids and plasmas. From the classical works of Kelvin and Helmholtz to the modern insights of Arnold, Moffatt, among others, topology plays a central role in understanding fluid motion. Quantities such as helicity and enstrophy encode topological constraints and capture fine-scale structures, including turbulent energy cascades. These concepts are crucial for addressing challenges such as solar corona heating and the computation of plasma equilibria. We discuss the role of topology preservation in the long-term simulation of fluids and plasmas, including processes such as magnetic relaxation and dynamo action.

While discrete differential forms have achieved great success in areas ranging from computational electromagnetism to fluid mechanics, many important problems in continuum mechanics and differential geometry fundamentally involve tensor fields. To this end, we develop Finite Element Tensor Calculus (FETC), using the Bernstein–Gelfand–Gelfand (BGG) machinery to extend de Rham complexes to tensor-valued quantities such as strain, stress, curvature, and torsion. A new finite element periodic table emerges, unifying discrete Regge metrics, distributional curvature, de Rham currents, and intrinsic finite element formulations in continuum mechanics.

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