COLLOQUIUM

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A Dual-space Multilevel Kernel-splitting Framework for Discrete and Continuous Convolution

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ABSTRACT

We introduce a new class of multilevel, adaptive, dual-space methods for computing fast convolutional transforms. These methods can be applied to a broad class of kernels, from the Green's functions for classical partial differential equations (PDEs) to power functions and radial basis functions such as those used in statistics and machine learning. The DMK (dual-space multilevel kernel-splitting) framework uses a hierarchy of grids, computing a smoothed interaction at the coarsest level, followed by a sequence of corrections at finer and finer scales until the problem is entirely local, at which point direct summation is applied.

The main novelty of DMK is that the interaction at each scale is diagonalized by a short Fourier transform, permitting the use of separation of variables, but without requiring the FFT for its asymptotic performance. The DMK framework substantially simplifies the algorithmic structure of the fast multipole method (FMM) and unifies the FMM, Ewald summation, and multilevel summation, achieving speeds comparable to the FFT in work per gridpoint, even in a fully adaptive context. For continuous source distributions, the evaluation of local interactions is further accelerated by approximating the kernel at the finest level as a sum of Gaussians with a highly localized remainder. The Gaussian convolutions are calculated using tensor product transforms, and the remainder term is calculated using asymptotic methods. We illustrate the performance of DMK for both continuous and discrete sources with extensive numerical examples in two and three dimensions.

This is joint work with Leslie Greengard.

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