THE UNIVERSITY OF CHICAGO | THE COMMITTEE ON COMPUTATIONAL AND APPLIED MATHEMATICS

COLLOQUIUM

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Gaussian Kernelized Graph Laplacian: Eigen-Convergence and Bi-Stochastic Normalization

TUESDAY, March 28th, at 4:00 PM
Jones 303, 5747 S. Ellis Ave. Chicago, IL 60637

ABSTRACT

Eigen-data of graph Laplacian matrices are widely used in data analysis and machine learning, for example, dimension reduction by spectral embedding. Under the manifold data setting, a kernelized graph affinity matrix is constructed from N data points i.i.d. sampled from a general unknown d-dimensional manifold embedded in a possibly high-dimensional space. The convergence of the graph Laplacian matrix to the manifold Laplacian operator in the large sample limit is a fundamental question, the studies of which dated back to the original work of Laplacian Eigenmap by Belkin-Niyogi in the 2000s. In this talk, we first introduce a result of the eigen-convergence (namely the convergence of eigenvalues and eigenvectors) of graph Laplacians to the Laplace-Beltrami operator, where choosing a smooth kernel function contributes to the improvement of convergence rates from previous results. By analyzing the Dirichlet form convergence and constructing candidate approximate eigenfunctions via convolution with manifold heat kernel, we prove eigen-convergence with rates as N increases and the kernel bandwidth ϵ decreases accordingly. When data density is non-uniform on the manifold, we prove the same rates for the density-corrected graph Laplacian. Time permitting, we introduce the use of bi-stochastic normalization to improve the robustness of graph Laplacian to high-dimensional outlier noise with a proven convergence guarantee. The analysis suggests an approximate matrix scaling problem that can be solved by Sinkhorn iterations with early termination. We show numerical experiments to support the theory.

Joint work with Nan Wu and Boris Landa.

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