



THE UNIVERSITY OF  
**CHICAGO**

THE COMMITTEE ON  
COMPUTATIONAL AND  
APPLIED MATHEMATICS

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JEFFREY ZHANG

Department of Mathematical Sciences  
Carnegie Mellon University

## On Local Minima in Polynomial Optimization

THURSDAY, March 10, 2022 at **3:30pm**  
Jones 303, 5747 S. Ellis Ave. Chicago, IL 60637  
OR  
via Zoom

We settle the computational complexity of some fundamental questions about local minima in polynomial optimization. We show that unless  $P=NP$ , there cannot be a polynomial-time algorithm that finds a point within Euclidean distance  $c^n$  (for any constant  $c$ ) of a local minimum of an  $n$ -variate quadratic program. This result answers a question of Pardalos and Vavasis that appeared in 1992 on a list of seven open problems in complexity theory for numerical optimization. By contrast, through leveraging techniques from algebraic geometry, we show that a local minimum of a cubic polynomial can be found efficiently by semidefinite programming (SDP), despite the fact that their critical points are NP-hard to find. We prove that second-order points of cubic polynomials admit an efficient semidefinite representation, which produces an efficiently-checkable necessary and sufficient condition for local minimality of a point for a cubic polynomial.

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Organizer:

Yuehaw Khoo, Department of Statistics (CAMI), [ykhoo@uchicago.edu](mailto:ykhoo@uchicago.edu)  
CAM Colloquium URL: <https://cam.uchicago.edu/events/cam-colloquium/>

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