THE UNIVERSITY OF CHICAGO

COMPUTATIONAL AND APPLIED MATHEMATICS COLLOQUIUM

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Solutions to the Minimum Variance Problem and Triangulation

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ABSTRACT

In moment closure approximations it is essential to understand the range of admissible higher order moments given the lower order moments and the support of the process. In some approximations, errors arise from underestimation of the variance in a hidden distribution given its mean. To avoid these errors one must solve a minimum variance problem (MVP): given a finite or countable set of points Ω in $\mathbb{R}^n$, a fixed mean $\bar{x} \in \text{conv}(\Omega)$, and a norm ρ on symmetric positive semi-definite matrices, find the space of distributions p supported on Ω with mean $\bar{x}$ that minimize $\rho(\mathbb{V}_p[X])$ where $\mathbb{V}_p[X]$ is the covariance of the distribution. We show that, for appropriately chosen norms, the support of all solutions to the MVP is concentrated at points near $\bar{x}$, and for a broad family of norms the solution is unique. We then show that, for $\rho(V) = \trace(V)$, the MVP is a linear programming problem and the space of all solutions is a polytope whose extreme solutions are associated with the tent functions on a Delaunay triangulation of Ω. We conclude by discussing properties of solutions for the induced two-norm.

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