The Bayesian inference step at the core of filtering algorithms is challenging for high-dimensional and non-Gaussian state-space models. While ensemble Kalman filters (EnKF) can yield robust estimates of the state in many practical systems, these methods are limited by linear transformations and are generally inconsistent with the Bayesian solution in the large-sample limit. In this presentation, I will discuss how measure transport can be used to construct couplings that map a prior ensemble into a collection of posterior samples. This approach can be understood as a natural generalization of the EnKF to nonlinear updates, and can reduce the intrinsic bias of the EnKF with a marginal increase in computational cost. In small-sample settings, we show to robustly learn these maps under sparse and low-rank structural assumptions and we demonstrate their benefit for filtering applications in chaotic dynamical systems and turbulent flow estimation. Finally, we comment on the broader utility of these algorithms in the setting of simulation-based inference where the likelihood function or prior is intractable to evaluate.